

Exercises: The numbers in the test problems will probably be simpler, but the format will be the same.

1. Find the exact distance between $\left(\frac{2}{5}, \frac{4}{3}\right)$ and $\left(\frac{4}{3}, \frac{3}{2}\right)$.

a. $\sqrt{\frac{253}{300}}$

b. $\sqrt{\frac{475}{250}}$

c. $\sqrt{\frac{253}{150}}$

d. $\sqrt{\frac{243}{300}}$

ae. $\sqrt{\frac{257}{300}}$

be. $\sqrt{\frac{253}{30}}$

ce. $\sqrt{\frac{253}{200}}$

de. $\sqrt{\frac{153}{300}}$

2. Find the distance between (2,3) and (5,-4). Round to the nearest thousandth.

a. 7.612

b. 7.614

c. 7.616

d. 7.618

ae. 7.620

be. 7.622

ce. 7.624

de. 7.626

3. Find the exact midpoint of the line segment between (1,7) and (3,-2)

a. $\left(2, \frac{5}{2}\right)$

b. $\left(2, \frac{3}{2}\right)$

c. $\left(2, \frac{5}{4}\right)$

d. $\left(\frac{2}{3}, \frac{5}{2}\right)$

ae. $\left(\frac{2}{7}, \frac{5}{2}\right)$

be. $\left(2, \frac{1}{2}\right)$

ce. $\left(\frac{2}{3}, \frac{3}{2}\right)$

de. $\left(\frac{2}{7}, \frac{1}{2}\right)$

4. Find the exact midpoint of the line segment between (-2,7) and (5,2). Round to the nearest hundredth.

a. (1.5,4.5)

b. (2.5,4.5)

c. (1.5,5.5)

d. (2.5,5.5)

ae. (1.5,2.5)

be. (2.5,3.5)

ce. (1.5,2.5)

de. (1.5,3.5)

Answers:

$$1. \sqrt{\left(\frac{4}{3} - \frac{2}{5}\right)^2 + \left(\frac{3}{2} - \frac{4}{3}\right)^2} = \sqrt{\left(\frac{5 \cdot 4 - 2 \cdot 3}{3 \cdot 5}\right)^2 + \left(\frac{3 \cdot 3 - 2 \cdot 4}{2 \cdot 3}\right)^2} = \sqrt{\left(\frac{14}{15}\right)^2 + \left(\frac{1}{6}\right)^2}$$

$$= \sqrt{\left(\frac{14}{15}\right)^2 + \left(\frac{1}{6}\right)^2} = \sqrt{\frac{196}{225} + \frac{1}{36}} = \sqrt{\frac{6831}{8100}} = \sqrt{\frac{253}{300}} \quad \text{a.}$$

$$2. \sqrt{(5-2)^2 + (-4-3)^2} = \sqrt{9+49} = \sqrt{58} \approx 7.61577 \quad \text{c.}$$

3. a

4. a

*Explanations – why do these computations work?***Distance**

When we have two points in the xy -plane as shown. The Pythagorean Theorem tells us that the distance between the points (the length of the hypotenuse) is

$$\text{given by } d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}.$$

Because $x_1 - x_2 = -(x_2 - x_1)$ and

$(x_1 - x_2)^2 = (x_2 - x_1)^2$, there are several ways to calculate this value if you want to avoid subtracting a negative number from another negative number.

Ex 1: Find the exact distance between $(1,7)$ and $(4,12)$.

Soln: The correct answer is $\sqrt{(4-1)^2 + (12-7)^2} = \sqrt{34}$.

$\sqrt{34} \approx 5.830951895\dots$ is *not* correct because it is not exact.

Midpoint

The midpoint of the line segment between $P_1 = (x_1, y_1)$ and $P_2 = (x_2, y_2)$ is found by averaging the x -coordinates and averaging the y -coordinates. So $M = \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2} \right)$. There is also a geometrical derivation of this, which uses proportional triangles associated with lines transecting three parallel lines. The idea of averaging is simpler.

