Functions
Single-Valued Relations

Last updated 1/27/2012

Exercises:

1. Which of the completely tabulated relations at the right give(s) a function y of x?

a

x	**y**
2	3
5	5
4	4
2	1

b

x	**y**
5	3
3	5
4	4
1	3

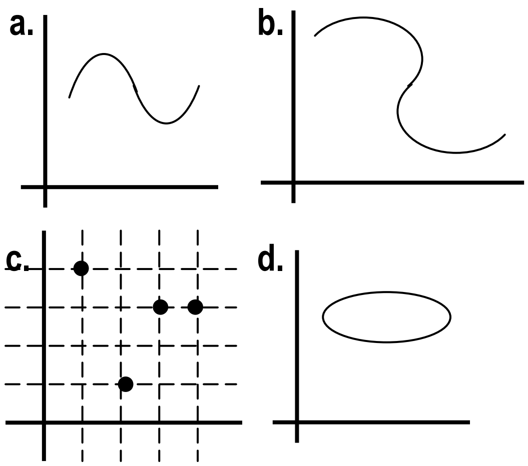
c

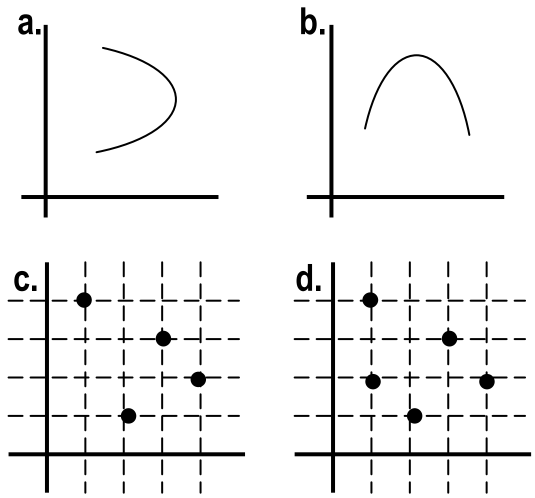
x	**y**
3	4
5	2
1	3
5	4

d

x	**y**
3	7
6	4
4	3
2	5

2. Which of the graphs below represent(s) a relation where y is a function of x?





3. Which of the graphs at the right represent(s) a relation where y is a function of x?

4. Which of the equations below represent(s) a relation where y is a function of x?

a. $y=\dfrac{-x\pm\sqrt{5x-2}}{3}$ **b**. $y=\dfrac{-x-\sqrt{5x-2}}{3}$ **c**. $7x^{12}-16y^{9}=1$

d. $6x^{13}-15y^{26}=1$ **e**. $y=\pm\sqrt{x}$

5. For $f(x)=\dfrac{2}{3x-5}$ write

a. f(4) as a reduced fraction **b**. f(a) **c**. f(a + h) **d.** f(x + h)

6. For $f(x)=7\sqrt{5x+3}$ write

a. f(4) simplified **b**. f(a) **c**. f(a + h) **d.** f(x + h)

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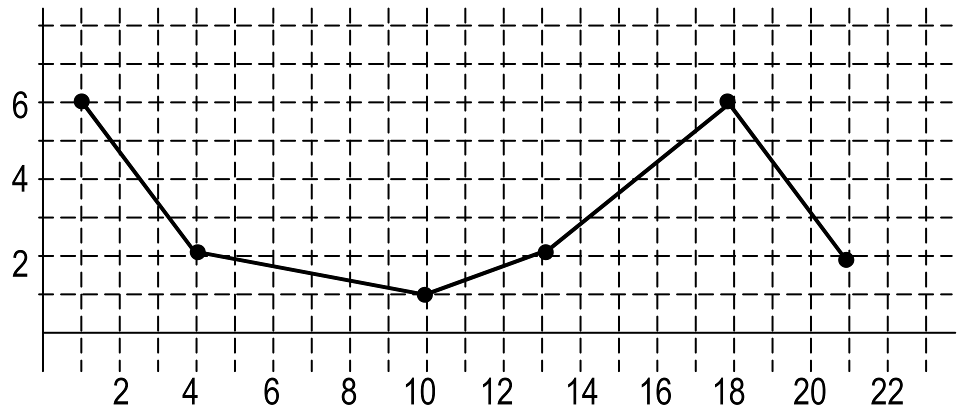
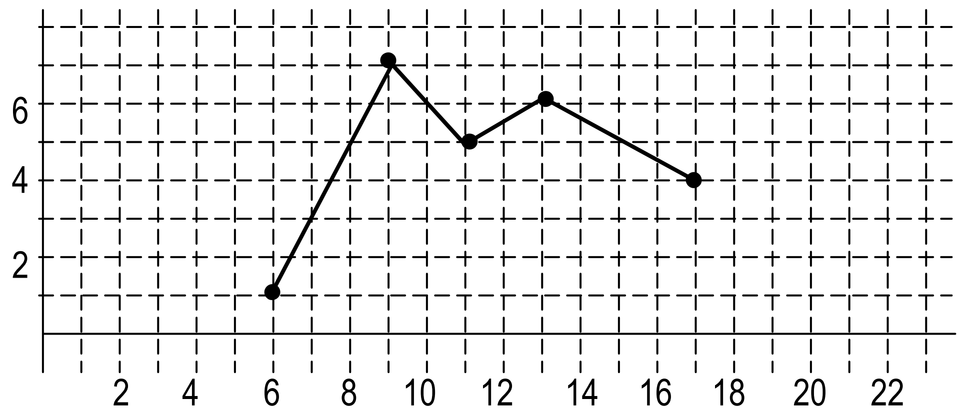
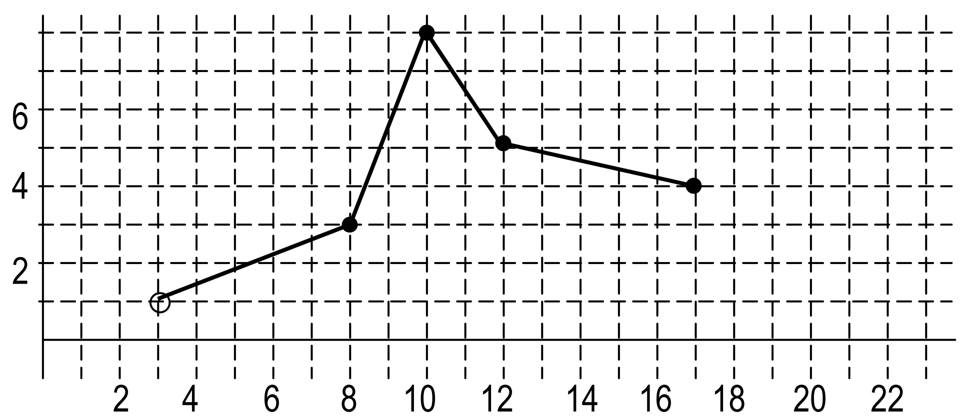
7. Give the domain and range of the function graphed at the right..



8. Give the domain and range of the function graphed at the right..



9. Give the domain and range of the function graphed at the right.



10. Write (without simplifying) $\frac{f(x+h)-f(x)}{h}$ for

a. $f(x)=3x^2-9$

b. $f(x)=\frac{2}{3x-5}$, for $x \neq 5/3$

c. $f(x)=\frac{5}{3x+2}$, for $x \neq -2/3$

d. $f(x)=\sqrt{x-7}$, for $x \geq 7$

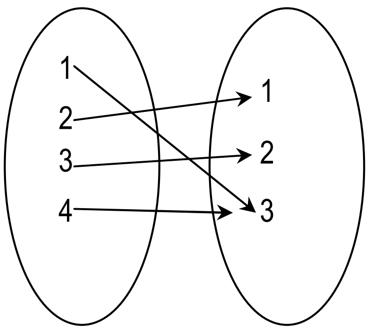
e. $f(x)=\sqrt{3x+2}$, for x $\geq$ -2/3
f. $f(x)=2\sqrt{7x-3}$, for x $\geq$ 3/7
g. $f(x)=2\sqrt{8-x}$, for x $\leq$ 8
h. $f(x)=7\sqrt{5-3x}$, for x $\leq$ 5/3

11. Give the domain and range of the function listed at the right.

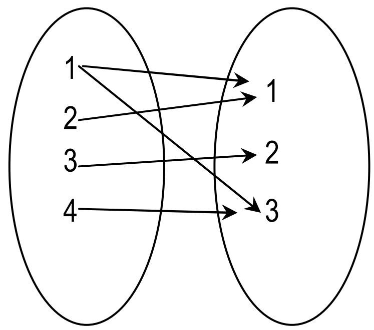
x	y
1	2
3	5
2	2
7	5

12. Which of the following represents a function y of x?

a.



b.



Answers:

1. b,d

2. a,c

3. b,c

4. b,c

5.

a. 2/7 **b.** $\frac{2}{3a-5}$ **c.** $\frac{2}{3(a+h)-5}$ **d.** $\frac{2}{3(x+h)-5}$

6.

a . $7\sqrt{23}$ **b.** $7\sqrt{5a+3}$ **c.** $7\sqrt{5(a+h)+3}$ **d.** $7\sqrt{5(x+h)+3}$

7. Domain [1, 21] Range [1, 6]

8. Domain [6, 17] Range [1, 7]

9. Domain (3,17] Range (1,8]

10.

a. $\frac{f(x+h)-f(x)}{h}=\frac{3(x+h)^2-9-(3x^2-9)}{h}$

b. $\frac{f(x+h)-f(x)}{h}=\frac{\frac{2}{3(x+h)-5}-\frac{2}{3x-5}}{h}$, $x \neq 5/3$

c. $\frac{f(x+h)-f(x)}{h}=\frac{\frac{5}{3(x+h)+2}-\frac{5}{3x+2}}{h}$, $x \neq -2/3$

d. $\frac{f(x+h)-f(x)}{h}=\frac{\sqrt{(x+h)-7}-\sqrt{x-7}}{h}$, $x > 7$

e. $\frac{f(x+h)-f(x)}{h}=\frac{\sqrt{3(x+h)+2}-\sqrt{3x+2}}{h}$

f. $\frac{f(x+h)-f(x)}{h}=\frac{2\sqrt{7(x+h)-3}-2\sqrt{7x-3}}{h}$

g. $\frac{f(x+h)-f(x)}{h}=\frac{2\sqrt{8-(x+h)}-2\sqrt{8-x}}{h}$, for $x \leq 8$

h. $\frac{f(x+h)-f(x)}{h}=\frac{2\sqrt{5-3(x+h)}-2\sqrt{5-3x}}{h}$, for $x \leq 5/3$]

11. Domain = {1, 3, 2, 7} Range = {2, 5}

12. a

Understanding the problems

Algebraic relations are generally described by equations and inequalities. We prefer situations where we can refer to "***the*** value of y when x is 7". In other words we have a particular interest in a *single-valued relation*. Such a relation is called a *function*.

Aside from this connection to definite noun phrases, we use functions and relations as mathematical models of various systems in physics, chemistry, biology and psychology. Traditionally we emphasize systems where specifying an input *determines* the output. These are called *determinate systems*. They are represented by functions.

Vertical Line Test

Think of how you read a graph to find a y-value. As in Fig. 1, from a point on the x-axis, run vertically to reach the graph, and then run horizontally to reach the y-axis.

Fig 1

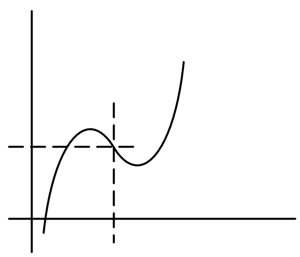
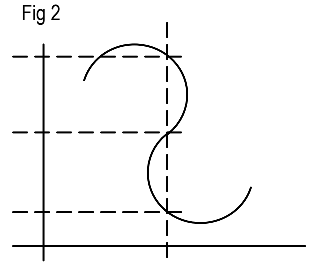


Fig 2

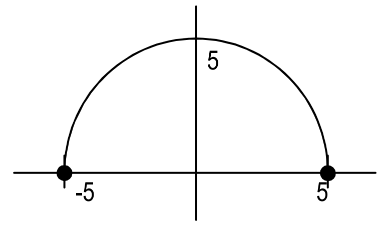


If some vertical line touches the graph more than once, , as in Fig 2, you would find more than one y-value, so the relation would not be single-valued. This gives the following.

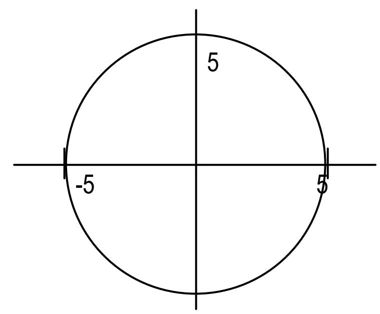
The Vertical Line Test: *If the graph of a relation is touched by some vertical line more than once, then that relation is not a function.*

Single-valuedness in Equations

In the graph shown for the upper half of a circle, $y = \sqrt{5^2 - x^2}$. The value of y depends on the value of x. The variable y is called the *dependent variable*, or *output*, while x is called the *independent variable*, or *input*.



If we were to describe the full circle, letting the formula become $y = \pm\sqrt{5^2 - x^2}$, y would no longer be a function of x because, for example, when x = 3, y = $\underline{+}$4.



When we see equations like $x^2 + y^2 = 5^2$, where y is not isolated. We can sometimes solve the equation carefully to get $y = \pm\sqrt{5^2 - x^2}$ and see that y is not a function of x. This happens when the exponent on y in the original equation is even. The critical fact is that $(-4)^2 = 16 = (+4)^2$.

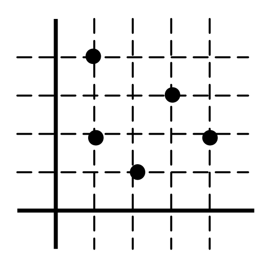
In an equation like $x^8 + y^9 = 5^2$, we can solve to get $y = \sqrt[9]{5^2 - x^8}$. Here we do *not* need $y = \pm\sqrt[9]{5^2 - x^8}$ since $(-4)^9$ is negative while $(+4)^9$ is positive. This gives the following conclusion.

Relations described by inequalities like y > x + 7 *never* give y as a single-valued function of x.

The Output Exponent Test: *In simple equations, where each variable x and y occurs just once, y is a function of x if the exponent of y is odd.*

Functions and Data Tables

Thinking about how we read function values from a graph, we can see that the points (1,2) and (1,4) are both on the graph of the relation at the right. In a tabulation of that relation we will see the lines highlighted below.



x	1	1	2	3	4
y	2	4	1	3	2

MAC 1105 *Functions* *Instr: Jamieson*

Chap 1 Sec 3 *Single-Valued Relations*

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The Repeated Input Test: *If a data table has a repeated input value (and the repetitions have different output values), then the tabulated relation is not a function.*

Ex1: For which of these (*completely tabulated relations*) is y a function of x?

a.

x	**y**
2	4
5	7
4	6
2	3

b.

x	**y**
2	3
7	6
5	4
3	3

c.

x	**y**
2	5
4	1
2	5
5	6

Soln: For **a.**, if x = 2 then y can be 4 or 3, so y is not a function of x.

In **b.**, there is no repeated x value, so y is a function of x.

In **c.**, technically if x = 2 then y must be 5. So mathematicians generally say y a function of x here. In statistics, these two lines in the table might represent two individuals with matching characteristics, which should possibly be considered separately, not as the same point. I will not give you such an example on a test – just clear cases like **a.** and **b.**

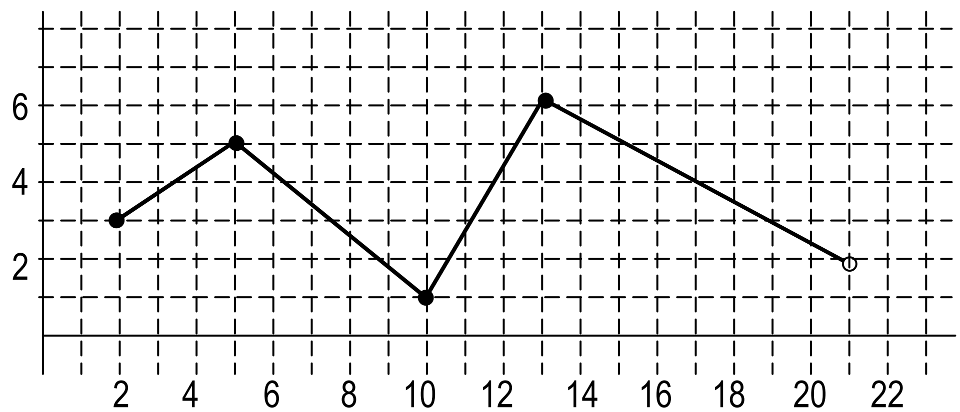
The domain of a function *is the set of all possible input values.*

The range of a function *is the set of all possible output values.*

Ex: Give the domain and range. Use interval notation.

Domain: [2, 21)

Range: [1, 6]



For the semi-circle discussed above, the domain is the interval [-5,5] and the range is the interval [0,5]. Both the domain and range are usually evident in a graph. In the circle, the domain is the interval [-5,5] and the range is also [-5,5].

Ex 2: Give the domain of each of the following functions. The range of a function is not generally obvious from a formula.

a. $y = \frac{5}{x}$ **b.** $y = \sqrt{x}$ **c.** $y = \sqrt{2x+5} + x^3$ **d.** $y = \frac{x^5 - 7}{4x+3}$

Remember that an expression ina denominator cannot be 0, and an expression under a square root (or other radical with even index) must be positive or 0, that is it must be ≥ 0

Soln:

a. $x \neq 0$

b. $x \geq 0$

c. $2x + 5 \geq 0$ $2x \geq -5$ $x > \frac{-5}{2}$

Do not just say "$x \geq 0$" whenever you see a radical. Word association is not mathematical reasoning. Make a statement about the expression under the radical, and simplify it to a statement about x.

Notice that x^3 is always defined so it does not affect the domain.

d. $4x + 3 \neq 0$ $\quad$ $4x \neq -3$ $\quad$ $x \neq \dfrac{-3}{4}$

Again, do not just say "$x \neq 0$" when you see x in a denominator. Also understand that $x^5 - 7$ is always defined, and it is allowed to be zero, so it does not affect the domain.

We just need to express the restriction $4x + 3 \neq 0$ and simplify it.

Function Notation

When y is a function of x, we often give the function a name to use as a short hand. We often use the letter f s a function name, or g or h. If $y = 3x^2 + 4x - 7$, then we define a function expression $f(x) = 3x^2 + 4x - 7$. We read this as "f of x" or sometimes "f at x". We may write f(5). This does *not* mean to multiply the f expression times 5 – instead it means to replace each "x" with a 5.

So $\quad f(5) = 3\cdot5^2 + 4\cdot5 - 7 = 88$.

Function Notation f(x) *tells us that when we have some algebraic expression M, then we form the expression f(M) by replacing each x with (M).*

In slightly more complex instances, which are common and very important, you may see expressions such as f(3 + h). This has the value $3(3 + h)^2 + 4(3 + h) - 7$.

Notice that if you do not include parentheses around 3 + h, then you have

$3\cdot3 + h^2 + 4\cdot3 + h - 7$. That is not at all the same – not even close. Your calculator will give you an incorrect answer if you put in something like 9 for h.

The issue here is whether you understand the order of operations – "Please Excuse My Dear Aunt Sally", or PEMDAS – and this is one of the major concerns of the Math Department at CF.