Linear Functions and Slope

Exercises:

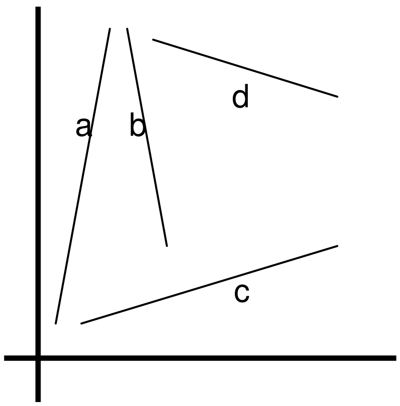
1. Match:

m = 0.3

m = -0.3

m = -7

m = 7



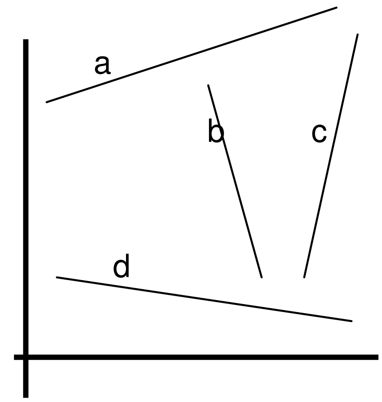
2. Match

m = 0.3

m = -0.3

m = -7

m = 7



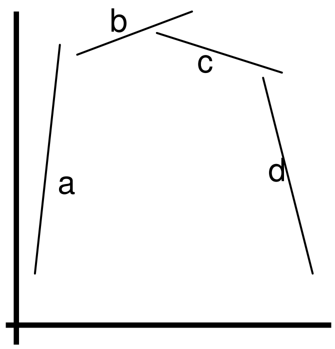
3. Match

m = 0.3

m = -0.3

m = -7

m = 7



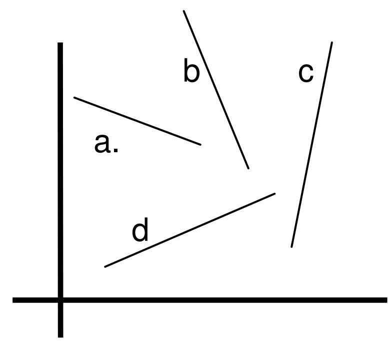
4. Match

m = 0.3

m = -0.3

m = -7

m = 7



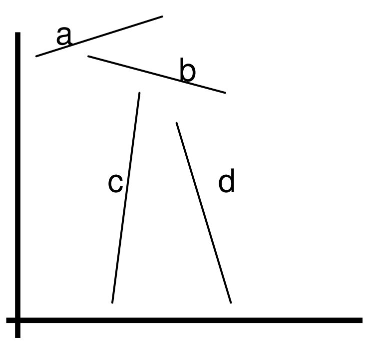
5. Match

m = 0.3

m = -0.3

m = -7

m = 7



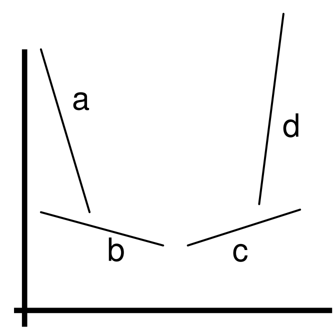
6. Match

m = 0.3

m = -0.3

m = -7

m = 7



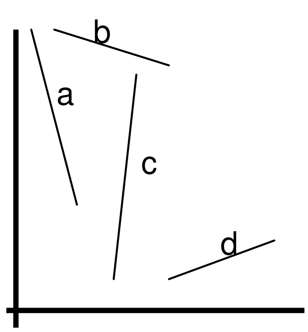
7. Match

m = 0.3

m = -0.3

m = -7

m = 7



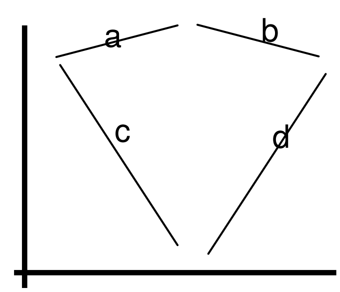
8. Match

m = 0.3

m = -0.3

m = -7

m = 7



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Linear Functions and Slope

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9. For each of these linear functions.
give the slope m = ______
the x-intercept (______,______)
and
the y-intercept (______,______)

a.

x	y
-4	12
-2	8
0	4
2	0
4	-4
6	-8
8	-12
10	-16
12	-20
14	-24

b.

x	y
-12	-16
-8	-8
-4	0
0	8
4	16
8	24
12	32
16	40
20	48
24	56

c.

x	y
-4	-1.4
-2	-0.7
0	0
2	0.7
4	1.4
6	2.1
8	2.8
10	3.5
12	4.2
14	4.9

d.

x	y
2	15
3	12
4	9
5	6
6	3

10. Classify each function below as constant, linear non-constant, or non-linear.

a. $y = \frac{3}{5x+7}$

b. $y = \frac{1}{3}(5x+7))$

c.

x	y
5	5
7	5
9	5
11	5

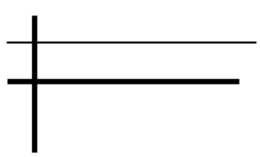
d.

x	y
2	2
7	4
12	8
17	12

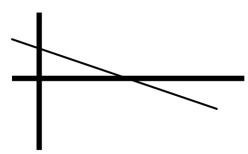
e.

x	y
3	8
6	6
9	4
12	2

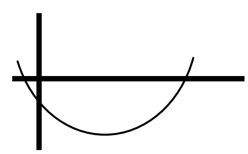
f.



g.



h.



11. Write the difference quotient $\frac{f(x+h)-f(x)}{h}$ for each function f(x) and simplify it until you have divided out the h in the denominator.

a. $f(x) = 4x^2 - 3x - 2$
b. $f(x) = 3x^2 + 7x - 5$
c. $f(x) = -2x^2 + 4x + 3$
d. $f(x) = 5x^2 + 7x - 1$
e. $f(x) = x^2 - x + 13$
f. $f(x) = 27x^2 - 5x - 7$
g. $f(x) = \frac{1}{x-3}$, x = 7
h. $f(x) = \frac{2}{x+5}$, x = 4
i. $f(x) = \frac{3}{2x-1}$, x = 4

j. $f(x)=\frac{5}{3x+7}$, x = 2

k. $f(x)=\frac{7}{5x-3}$, x = 4

l. $f(x)=\sqrt{x-1}$, x = 7

m. $f(x)=\sqrt{x+5}$, x = 2

n. $f(x)=\sqrt{5x+2}$, x = 3

o. $f(x)=\sqrt{2x-3}$, x = 7

p. $f(x)=\sqrt{3x+5}$, x = 4

10. Give the slope-intercept equation of the line through (3,5) parallel to 4x – 5y = 11.

11. Give the slope-intercept equation of the line through (5,3) parallel to 5x – 4y = 11.

12. Give the slope-intercept equation of the line through (7,2) with slope $\frac{3}{4}$

13. Give the equation of line parallel to 3x + 4y = 1 through (3,2).

Answers:

1. cdba
2. adbc
3. bcda
4. dabc
5. abdc
6. cbad
7. dbac
8. abcd
9. . **a**: m = -2; x-intercept (2,0) ; y-intercept (0,4)
b: m = 2; x-intercept (-4,0) ; y-intercept (0,8)
c: m = 0.35; x-intercept (0,0) ; y-intercept (0,0)
d. m = -3; x-intercept (7,0) ; y-intercept (0,18)
10. 10.

a. non-linear **b**. linear non-constant
c. constant **d**. non-linear **e**. linear non-constant
f. constant **g**. linear non-constant **h**. non-linear

11.

a. $\frac{4(x+h)^2-3(x+h)-2-(4x^2-3x-2)}{h}$

$=\frac{4(x^2+2xh+h^2)^2-3(x+h)-2-(4x^2-3x-2)}{h}$

$=\frac{4\cdot 2xh+4\cdot h^2-3h}{h}=4\cdot 2x+4\cdot h-3$

b. $\dfrac{3(x+h)^2+7(x+h)-5-(3x^2+7x-5)}{h}$

$=\dfrac{3(x^2+2xh+h^2)+7(x+h)-5-(3x^2+7x-5)}{h}$

$=\dfrac{3\cdot 2xh+3\cdot h^2+7h}{h}=3\cdot 2x+3\cdot h+7$

c. $\dfrac{-2(x+h)^2+4(x+h)+3-(-2x^2+4x+3)}{h}$

$=\dfrac{-2(x^2+2xh+h^2)+4(x+h)+3-(-2x^2+4x+3)}{h}$

$=\dfrac{-2\cdot 2xh-2\cdot h^2+4h}{h}=-2\cdot 2x-2\cdot h+4$

d. $\dfrac{5(x+h)^2+7(x+h)-1-(5x^2+7x-1)}{h}$

$=\dfrac{5(x^2+2xh+h^2)+7(x+h)-1-(5x^2+7x-1)}{h}$

$=\dfrac{5\cdot 2xh+5\cdot h^2+7h}{h}=5\cdot 2x+5\cdot h-1$

e. $\dfrac{(x+h)^2-(x+h)+13-(x^2-x+13)}{h}$

$=\dfrac{(x^2+2xh+h^2)^2-(x+h)+13-(x^2-1x+13)}{h}$

$=\dfrac{2xh+h^2-h}{h}=2x+h-1$

f. $\dfrac{27(x+h)^2-5(x+h)-7-(27x^2-5x-7)}{h}=\dfrac{27(x^2+2xh+h^2)-5(x+h)-7-(27x^2-5x-7)}{h}$

$=\dfrac{27\cdot 2xh+27h^2-7h}{h}=27\cdot 2x+27h-7$

g. $\dfrac{\dfrac{1}{(7+h)-3}-\dfrac{1}{4}}{h}=\dfrac{4-(4+h)}{4(4+h)h}=\dfrac{-h}{4(4+h)h}=\dfrac{-1}{4(4+h)}$

h. $\dfrac{\dfrac{2}{(4+h)+5}-\dfrac{2}{9}}{h}=\dfrac{18-2(9+h)}{h(9+h)9}=\dfrac{-2h}{h(9+h)9}=\dfrac{-2}{(9+h)9}$

i. $\dfrac{\dfrac{3}{2(4+h)-1}-\dfrac{3}{7}}{h}=\dfrac{21-3(7+2h)}{(7+2h)7h}=\dfrac{-6h}{(7+2h)7h}=\dfrac{-6}{(7+2h)7}$

j. $\dfrac{\dfrac{5}{3(2+h)+7}-\dfrac{5}{13}}{h}=\dfrac{65-5(13+3h)}{(13+3h)13h}=\dfrac{-15h}{(13+3h)13h}=\dfrac{-15}{(13+3h)13}$

k. $\dfrac{\dfrac{7}{5(4+h)-3}-\dfrac{7}{17}}{h}=\dfrac{119-7(17+5h)}{h(17+5h)119}=\dfrac{-35h}{h(17+5h)119}=\dfrac{-35}{(17+5h)119}$

l. $f(x)=\dfrac{\sqrt{(7+h)-1}-\sqrt{6}}{h}\cdot\dfrac{\sqrt{6+h}+\sqrt{6}}{\sqrt{6+h}+\sqrt{6}}=\dfrac{(6+h)-6}{h\left(\sqrt{6+h}+\sqrt{6}\right)}=\dfrac{h}{h\left(\sqrt{6+h}+\sqrt{6}\right)}=\dfrac{1}{\sqrt{6+h}+\sqrt{6}}$

m. $\dfrac{\sqrt{(2+h)+5}-\sqrt{7}}{h}\cdot\dfrac{\sqrt{7+h}+\sqrt{7}}{\sqrt{7+h}+\sqrt{7}}=\dfrac{(7+h)-7}{h\left(\sqrt{7+h}+\sqrt{7}\right)}=\dfrac{h}{h\left(\sqrt{7+h}+\sqrt{7}\right)}=\dfrac{1}{\sqrt{7+h}+\sqrt{7}}$

n. $\dfrac{\sqrt{5(3+h)+2}-\sqrt{17}}{h}\cdot\dfrac{\sqrt{17+5h}+\sqrt{17}}{\sqrt{17+5h}+\sqrt{17}}=\dfrac{(17+5h)-17}{h\left(\sqrt{17+5h}+\sqrt{17}\right)}=\dfrac{5h}{h\left(\sqrt{17+5h}+\sqrt{17}\right)}=\dfrac{5}{\sqrt{17+5h}+\sqrt{17}}$

o. $\dfrac{\sqrt{2(7+h)-3}-\sqrt{11}}{h}\cdot\dfrac{\sqrt{11+2h}+\sqrt{11}}{\sqrt{11+2h}+\sqrt{11}}=\dfrac{(11+2h)-11}{h\left(\sqrt{11+2h}+\sqrt{11}\right)}=\dfrac{2h}{h\left(\sqrt{11+2h}+\sqrt{11}\right)}=\dfrac{2}{\sqrt{11+2h}+\sqrt{11}}$

p. $\dfrac{\sqrt{3(4+h)+5}-\sqrt{17}}{h}\cdot\dfrac{\sqrt{17+3h}+\sqrt{17}}{\sqrt{17+3h}+\sqrt{17}}=\dfrac{(17+3h)-17}{h\left(\sqrt{17+3h}+\sqrt{17}\right)}=\dfrac{3h}{h\left(\sqrt{17+3h}+\sqrt{17}\right)}=\dfrac{3}{\sqrt{17+3h}+\sqrt{17}}$

12. $y = 0.8x + 2.6$
13. $y = 1.25x – 3.25$
14. $y = 0.75x – 3.25$
15. $y = -0.75x + 4.25$

Explanations

A Linear Function *is one whose algebraic definition can be simplified to $y(x) = ax + b$. Graphically, linear functions are those whose graphs are straight lines.*

Formulas for linear functions cannot contain $|x|$, $\sqrt{x}$, x^2, x^3 or $\frac{1}{x}$. We will study these non-linear functions later. Also, an “x” in a linear formula cannot be multiplied directly or indirectly by another “x”.

For example, in $y = 5 – 2(3 – 7x) + 9$, x is only multiplied by constants.

Compare that with $y = 5 – 2x(3 – 7x) + 9$, where x is indirectly multiplied by another x, and the formula is equivalent to $y = 5 – 6x + 14x^2 + 9$

Consider a more challenging example.

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Chap. 1 Sec 4 | *Last updated 1/27/2012*

f(x) = 5x – 7[3x + 11(2x – 1) – 13(5 – 3x)] has several "x"s, some multiplied directly by constants and embedded in grouping symbols. Outside the grouping symbols there are other coefficients, but these are still just constants indirectly multiplied times x.

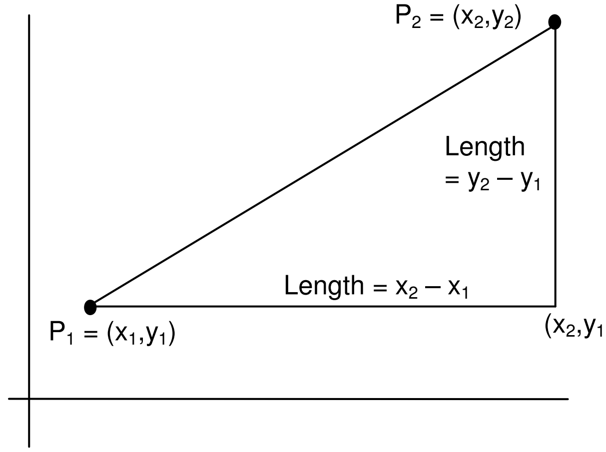
g(x) = 5x – 7[3x + 11(2x – 1) – 13x(5 – 3x)] is non-linear.

In the end, you can see that x is never multiplied by another x or subjected to any of the other (non-linear) functions you have seen in the past.

The slope m of the line segment between two points *$P_1 = (x_1,y_1)$ and $P_2 = (x_2,y_2)$ is the ratio of the rise over the run.*

Symbolically, $m = \dfrac{y_2 - y_1}{x_2 - x_1}$

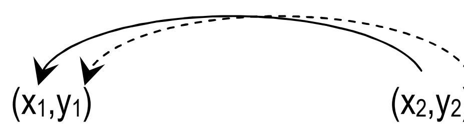
The rise and the run are signed quantities, positive or negative. The signs were irrelevant in computing distances because each quantity was squared. Now we need to be conscious of the sign.



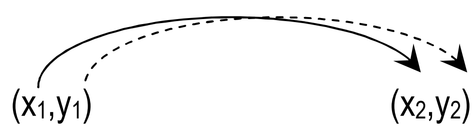
The line through these two points has many other segments all with the same slope

Ex: Find the slope of the line through (2,5) and (17,19).

Soln: $m = \dfrac{19-5}{17-2} = \dfrac{14}{5} = 2.8$ The decimal form is as good as the ratio form as long as you do not have to round to make a decimal.



In this computation we used (19,17) as (x_2,y_2), but nothing in the problem said to do that. If we calculate $m = \dfrac{5-19}{2-17} = \dfrac{-14}{-5} = \dfrac{14}{5} = 2.8$, we get the same result.



You need a ratio of differences, which may be taken in either direction, so long as the directions are the same.

The difference in the x coordinates is often abbreviated Δx,

while the difference in the y coordinates is Δy. Then the slope can be expressed as $\dfrac{\Delta y}{\Delta x}$.

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Linear Functions and Slope

Recall

1. A line with a positive slope rises as x increases.
2. A line with a negative slope falls as x increases
3. A line with 0 slope has constant height above the x-axis as x increases.
4. A vertical line such as the line between (3,5) and (3,12) has an undefined slope. In this example $\frac{\Delta y}{\Delta x}=\frac{7}{0}$.
5. Steep line compared with lines that are close to horizontal
a. A line which <u>*rises a lot*</u> as x increases has a <u>*larger positive slope*</u> than a line which rises just a little as x increases.
b. A line which <u>*falls a lot*</u> as x increases has a <u>*slope with larger absolute value*</u> than a line which falls just a little as x increases. While this line falls a lot and is steeper than a shallow line, its slope, as a real number is smaller. Be careful about that.

The content of **5 a,b** can be summarized as:
Lines which are close to horizontal have slopes close to zero
Lines which are steep to horizontal have slopes with large absolute value.

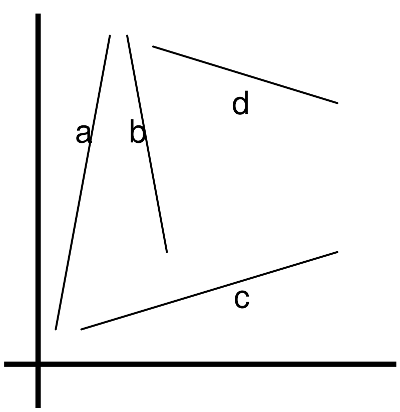
Ex 1
Match:

m = 0.3

m = -0.3

m = -7

m = 7



Soln 1:cdba

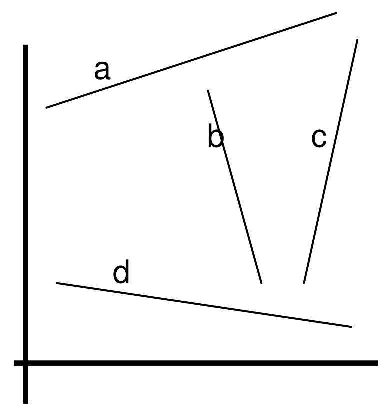
Ex 2
Match

m = 0.3

m = -0.3

m = -7

m = 7



Soln 2: adbc

Returning to the general characterizations of linearity, we have the following for tabulated functions.
A Linear Function *is one whose table shows the same change in output whenever the input changes by a specified amount*

Ex: Classify the following functions as constant, linear non-constant, or non-linear

a.

x	y
1	5
2	10
3	15
4	20

b.

x	y
2	213
4	213
6	213
8	213

c.

x	y
3	8
6	6
9	-4
12	2

d.

x	y
5	2
10	4
15	8
20	16

e.

x	y
2	10
4	100
6	1000
8	10000

Soln:
a. linear non-constant
b. constant
c. non-linear
d. non-linear
e. non-linear

Linear Functions and Slope

Finally, recall that in a graph or table, the y-intercept is the point where x = 0, and the x-intercept is the point where y = 0.