

Piecewise Defined Functions

Exercises:

1. Solve for x: $|3x + 17| = 0$

a. $\frac{17}{3}$

b. $\frac{-17}{3}$

c. $\frac{3}{17}$

d. $\frac{-3}{17}$

ae. $\frac{\pm 3}{17}$

be. $\frac{\pm 17}{3}$

ce. all reals

de. no solutions

2. Solve for x: $|2x + 17| = 5$

a. 6, 11

b. 6, -11

c. -6, 11

d. -6, -11

ae. all reals

be. no solutions

3. Solve for x: $|2x + 7| + 5 = 18$

a. 3, 10

b. 3, -10

c. -3, 10

d. -3, -10

ae. all reals

be. no solutions

4. Solve for x: $|2x + 7| + 18 = 5$

a. 0, 7

b. 0, 7

c. 0, 7

d. 0, 7

ae. all reals

be. no solutions

5. Solve for x: $|2x + 17| + 4 < 9$

a. $-11 < x < -6$

b. $-11 \leq x \leq -6$

c. $x < -11$ or $x > -6$

d. $x \leq -11$ or $x \geq -6$

ae. all reals

be. no solutions

6. Solve for x: $|2x + 17| + 4 \leq 9$

a. $-11 < x < -6$

b. $-11 \leq x \leq -6$

c. $x < -11$ or $x > -6$

d. $x \leq -11$ or $x \geq -6$

ae. all reals

be. no solutions

7. Solve for x: $|2x + 17| + 4 > 9$

a. $-11 < x < -6$

b. $-11 \leq x \leq -6$

c. $x < -11$ or $x > -6$

d. $x \leq -11$ or $x \geq -6$

ae. all reals

be. no solutions

8. Solve for x: $|2x + 17| + 4 \geq 9$

a. $-11 < x < -6$

b. $-11 \leq x \leq -6$

c. $x < -11$ or $x > -6$

d. $x \leq -11$ or $x \geq -6$

ae. all reals

be. no solutions

9. Solve for x: $|2x + 17| + 9 < 4$

a. $-11 < x < -6$

b. $-11 \leq x \leq -6$

c. $x < -11$ or $x > -6$

d. $x \leq -11$ or $x \geq -6$

ae. all reals

be. no solutions

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10. Solve for x: $|2x + 17| + 9 \leq 4$

a. $-11 < x < -6$

b. $-11 \leq x \leq -6$

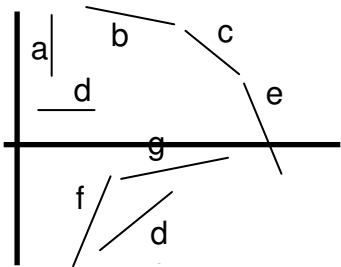
c. $x < -11$ or $x > -6$

d. $x \leq -11$ or $x \geq -6$

ae. all reals

be. no solutions

11. Match



___ $m = 3$

___ $m = -0.2$

___ $m = 1$

___ $m = -1.1$

___ $m = 0.3$

___ $m = -5$

___ $m = 0$

___ m is undefined

12. Match

___ $m = 3$

___ $m = -0.2$

___ $m = 1$

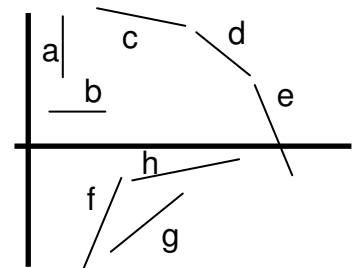
___ $m = -1.1$

___ $m = 0.3$

___ $m = -5$

___ $m = 0$

___ m is undefined



13. Match

___ $m = 3$

___ $m = -0.2$

___ $m = 1$

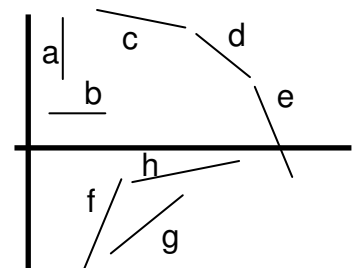
___ $m = -1.1$

___ $m = 0.3$

___ $m = -5$

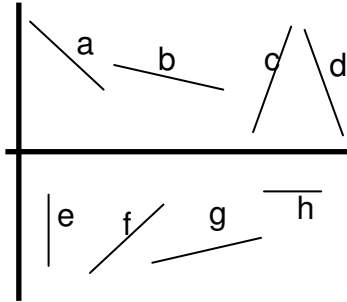
___ $m = 0$

___ m is undefined



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14. Match



___ $m = 3$

___ $m = -0.2$

___ $m = 0.9$

___ $m = -0.9$

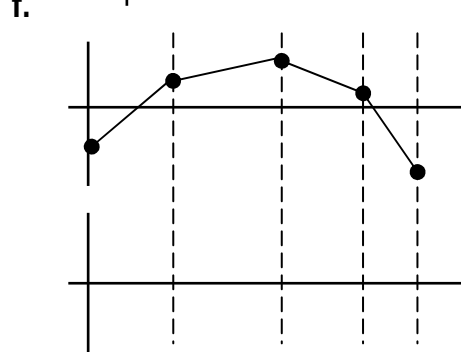
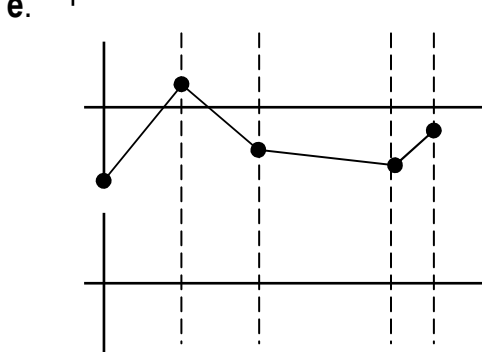
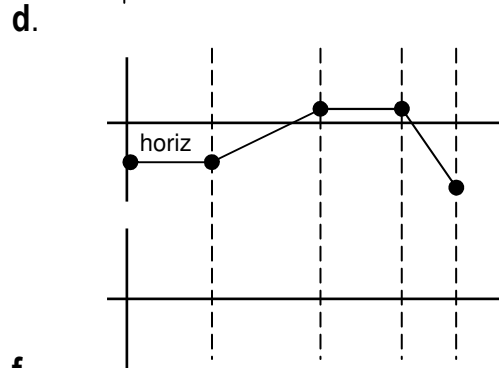
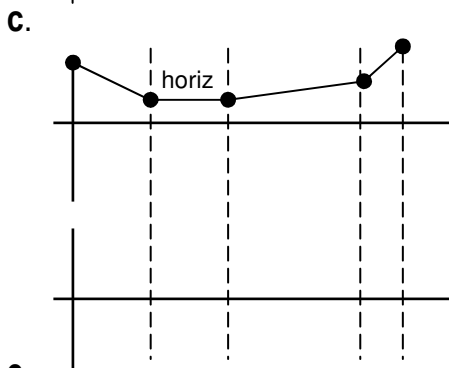
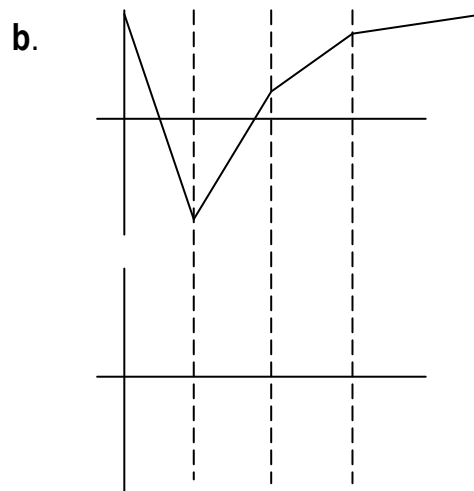
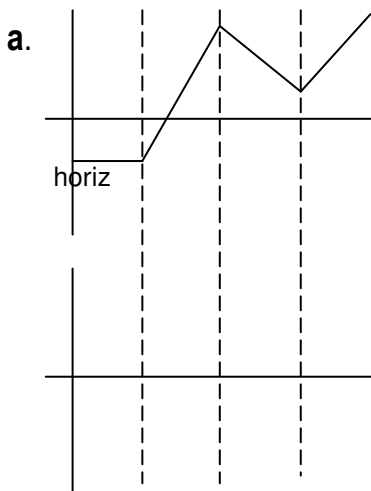
___ $m = 0.2$

___ $m = -3$

___ $m = 0$

___ m is undefined

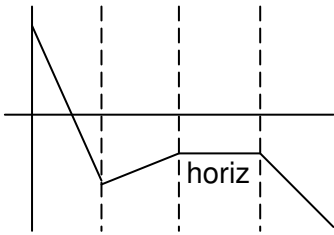
15. Sketch the slope function $m(x)$. Indicate where the slope is positive negative or zero and where it is increasing or decreasing.



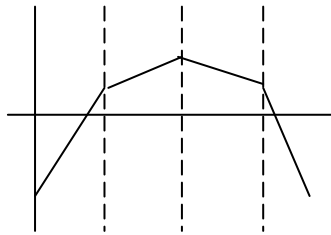
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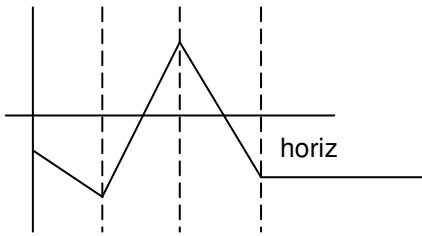
g.



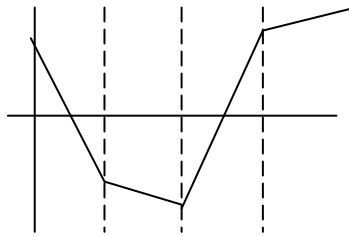
h.



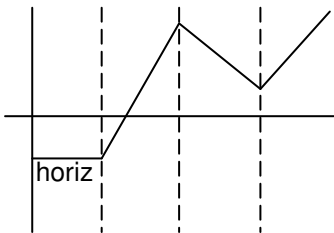
i.



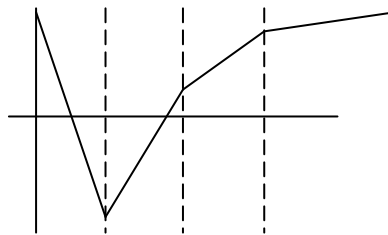
j.



k.



l.



16. For each function calculate the specified value:

$$\text{a. } f(x) = \begin{cases} 2x - 3 & , \text{if } x > 5 \\ x + 1 & , \text{if } x < 5 \\ 3 & , \text{if } x = 5 \end{cases}$$

$$f(3) + f(4) =$$

$$\text{b. } f(x) = \begin{cases} 2x + 3 & , \text{if } x < 5 \\ 7 & , \text{if } x = 5 \\ x - 1 & , \text{if } x > 5 \end{cases}$$

$$f(7) - f(5) =$$

$$\text{c. } f(x) = \begin{cases} 2x + 3 & , \text{if } x < 5 \\ 2x - 1 & , \text{if } x > 5 \\ 1 & , \text{if } x = 5 \end{cases}$$

$$f(1) + f(2) =$$

Answers:

1. b
2. d
3. b
4. be
5. a
6. b
7. c

- 8. d
- 9. be
- 10. be

11.

- f b
- h c
- g e
- d a

12.

- f c
- g d
- h e
- b a

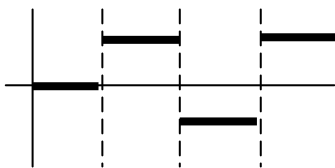
13. Top to bottom: c a e f g d b

14.

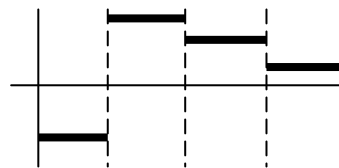
- f c
- g d
- h e
- b a

15.

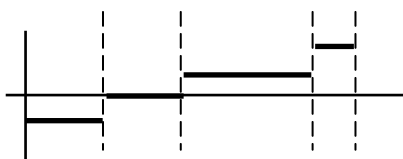
a.



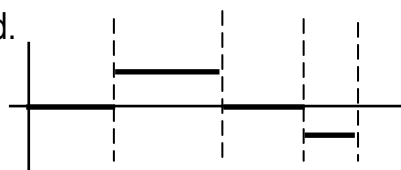
b.



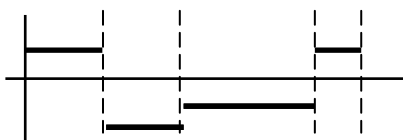
c.



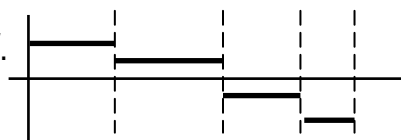
d.

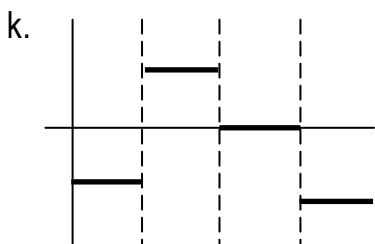
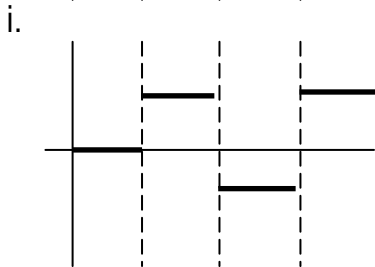
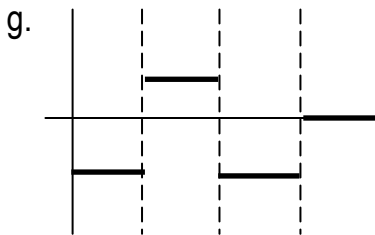


e.



f.



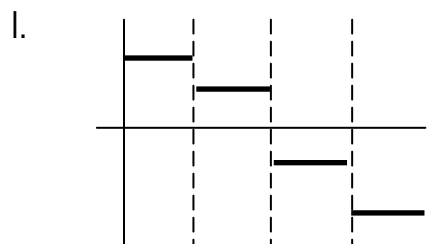
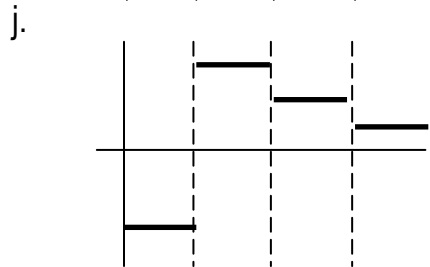
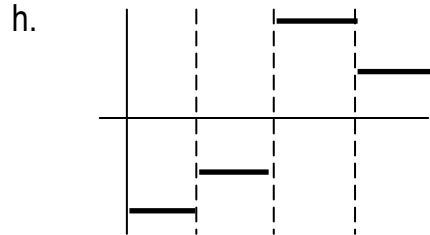


16.

a. 9

b. -1

c. 12



Understanding these ideas

In real world applications of mathematics, it is rare that a single formula models a relation for all time. Often an event occurs, like a highway being built near a small town or an employer coming to or leaving the town, which changes the growth trend of the town's population.

In these cases we piece together different formulas. "The population of the town" is still the quantity in question, so we do not say we have a different function.

Mathematicians describe this combination of functions as in the following example.

$$\mathbf{Ex\ 1:} \quad f(x) = \begin{cases} 2x + 1 & ,\text{if } x > 5 \\ 7 & ,\text{if } x = 5 \\ 12 - 3x & ,\text{if } x < 5 \end{cases}$$

This corresponds to the English statements

$$\text{if } x > 5, \text{ then } f(x) = 2x + 1$$

$$\text{if } x = 5, \text{ then } f(x) = 7$$

$$\text{if } x < 5, \text{ then } f(x) = 12 - 3x$$

Equivalently,

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$$f(x) = 2x + 1, \text{ if } x > 5$$

$$f(x) = 3, \text{ if } x = 5$$

$$f(x) = 12 - 3x, \text{ if } x < 5$$

A common mistake is to try to read the clauses of the mathematical expression from left to right, but you can see that is not something you can depend on even in plain English.

The most basic skill is evaluating these expressions:

$$f(4) = \underline{\hspace{2cm}}$$

$$f(5) = \underline{\hspace{2cm}}$$

$$f(6) = \underline{\hspace{2cm}}$$

$$f(7) = \underline{\hspace{2cm}}$$

Answers:

$$f(4) = 12 - 3 \cdot 4 = 0$$

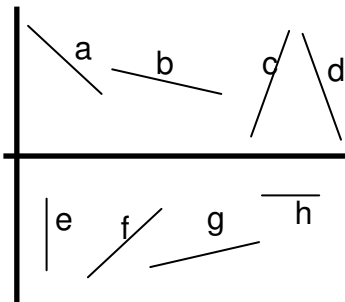
$$f(5) = 7, \text{ as given by the middle clause}$$

$$f(6) = 2 \cdot 6 + 1 = 13$$

$$f(7) = 2 \cdot 7 + 1 = 15$$

Our main interest in these functions is in developing an idea introduced in the following example.

Ex 2: Match each line segment to its approximate slope.



$$\underline{\hspace{1cm}} m = 3$$

$$\underline{\hspace{1cm}} m = -0.2$$

$$\underline{\hspace{1cm}} m = 0.9$$

$$\underline{\hspace{1cm}} m = -0.9$$

$$\underline{\hspace{1cm}} m = 0.2$$

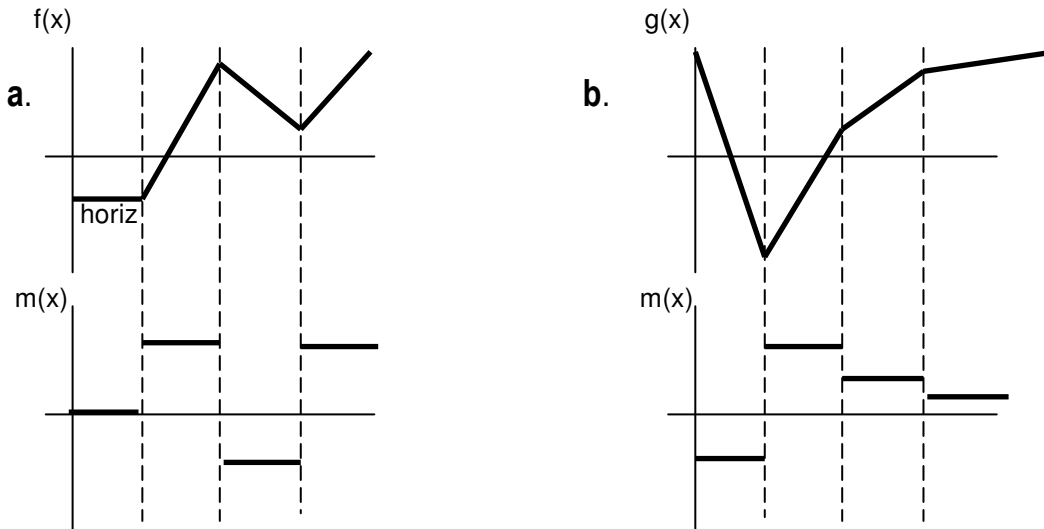
$$\underline{\hspace{1cm}} m = -3$$

$$\underline{\hspace{1cm}} m = 0$$

$$\underline{\hspace{1cm}} m \text{ is undefined}$$

Functions as shown in Ex 2 are called **piecewise linear**. For such functions the slope does not have a constant numerical value. The slope depends on which subinterval of the domain x is in. Therefore the slope is a function $m(x)$ depending on x .

Ex 3: Sketch the slope function $m(x)$, indicating where $m(x)$ is positive, negative, zero, increasing or decreasing.



Understand that “sketch” is a loose term. The directions do not call for precisions. The indications about positive negative and zero slope are clear enough.

“Increasing or decreasing” means that when two slope segments of the same sign are next to each other, you need to indicate which is higher. This does not happen in part a, but it does happen in part b.

We have some limited interest in graphing these piecewise functions from their algebraic definitions.

Ex 4: Sketch $f(x) = \begin{cases} 2x + 1 & , \text{if } x > 5 \\ 7 & , \text{if } x = 5 \\ 12 - 3x & , \text{if } x < 5 \end{cases}$ and determine if

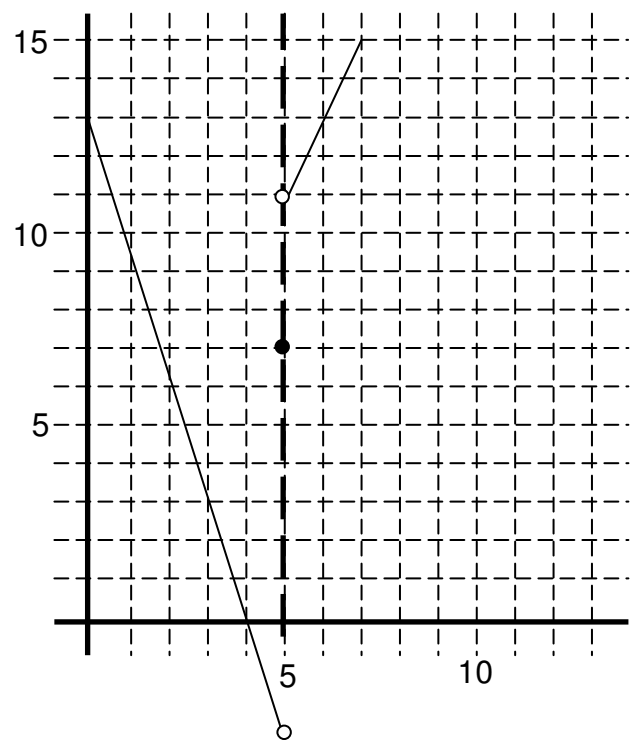
$f(x)$ is **continuous**. (Continuity is the property that a functions graph can be drawn without taking your pencil off the paper – the graph has only one piece.)

Solution:

It can be useful to draw a supplementary barrier line at the break value $x = 5$, as shown.

The clause corresponding to $x = 5$ gives a point $(5,7)$ on the graph.

The clauses for $x < 5$ and $x > 5$ do not give values for $x = 5$, so we can think of them as graphing to lines with open circles on the ends. However, one or both of their ends may coincide with the point $(5,7)$.



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Then we can graph the half lines (rays) given by the other two clauses. We might simply plot two points for each clause, or use the slope intercept understanding of the function description.

This function then is not continuous. If the two line converge on the same point, and that point is filled in by the third clause, then the function is continuous.

x	12 - 3x
0	12
4	0

x	2x + 1
6	13
7	15

Ex 5: Sketch $f(x) = \begin{cases} x + 2 & , \text{if } x > 5 \\ 7 & , \text{if } x = 5 \\ 12 - x & , \text{if } x < 5 \end{cases}$ and determine if $f(x)$ is continuous.

Solution:

Again we draw a supplementary barrier line at the break value $x = 5$.

The clause corresponding to $x = 5$ gives a point (5,7) on the graph.

We can graph the half lines given by the other two clauses using points in the table below.

x	12 - x
0	12
4	8

x	x + 7
6	7
7	8

This function then is continuous.

The Absolute Value Function

A particular case of piecewise definition is the absolute value function.

The absolute value of a number x is the distance from x to 0.

This is the geometrical definition of absolute value.

The algebraic definition is

The absolute value of x is $|x| = \begin{cases} x & , \text{if } x \geq 0 \\ -x & , \text{if } x < 0 \end{cases}$

