$$A_0\left(1+\frac{r}{n}\right)^{nt} \qquad A_oe^{rt}$$

1. I put $1500 in an account at 5% annual interest.
a. How long does it take to accumulate $2000 if the interest is compounded quarterly?
b. How long if the interest is compounded continuously?
c. What interest rate do I need to find to have $2000 in 4 years with continuously compounded interest?
d. What rate would I need if the interest is compounded quarterly?
2. The radioactive mass of a certain isotope decays exponentially. If the initial mass is 2 grams, and after 60 days the mass is 1.7 grams, give a specific model (formula) for the mass in grams at t days.
3. A population of bacteria grows exponentially. The initial population is 500. After 4 weeks, the population is 700. Give a specific model (formula) for the size of the population at t weeks.
4. A colony of mold grows exponentially from an initial mass of 1.7 grams to a mass of 3.5 grams after 6 weeks. Find a specific model for the mass in grams at t weeks.
5. The radioactive mass of a certain isotope decays exponentially. After 15 days the mass is 3 grams, and after 80 days the mass is 1.9 grams, give a specific model (formula) for the mass in grams at t days.
6. A population of bacteria grows exponentially. After 2 weeks the population is 200. After 4 weeks, the population is 500. Give a specific model (formula) for the size of the population at t weeks.
7. A colony of mold grows exponentially. At 2 weeks the mass is 1.3 grams. At 7 weeks the mass is 4.5 grams. Find a specific model for the mass in grams at t weeks.
8. The population of bacteria in a certain limited growth situation is
$P(t)=\dfrac{1.7\text{x}10^{17}}{1+(6.1\text{x}10^{9})e^{-0.57t}}$, where t is in days..
a. What is the limiting value for this population?
b. What is the population at 17 days?
9. A population grows logistically. Its limiting value is 20. Its initial size is 5. After 10 days its size is 16. Find its specific logistic model.
10. A farm lake is initially stocked with 10 fish. The food supply limits the population to 150. After 2 years the population is 40. Find a logistic model for the population.
11. A farm lake is initially stocked with 10 fish. The food supply limits the population to 150. Assuming a logistic model $P(t)=\dfrac{a}{1+be^{-kt}}$ for the population, find a and b.

12. A farm lake is initially stocked with a known number of fish. The food supply limits the population to known size. We partially determine a logistic model $P(t) = \frac{150}{1+14e^{-kt}}$ for the population. After 2 years the population is 40. Find k.

Answers:

1. 5.7895 yrs 5.7536 yrs 7.192% 7.257%
2. $m(t) = 2e^{-0.002708t}$ grams
3. $P(t) = 500e^{0.08411t}$
4. $m(t) = 1.7e^{0.12035t}$
5. $m(t) = 3.33348e^{-0.00702t}$
6. $P(t) = 80e^{0.4581t}$
7. $m(t) = 0.791107e^{0.24834t}$
8. limit = 1.7×10^{17}, $P(17) = 4.4522\times10^{11}$
9. $P(t) = \dfrac{20}{1+3e^{-0.24849t}}$
10. $P(t) = \dfrac{150}{1+14e^{-0.813728t}}$
11. a = 150, b = 14
12. k = -0.813728

More Problems with Some Answers:

13. The radioactive mass of a certain isotope decays exponentially. After 10 days the mass is 8.2 grams, and after 120 days the mass is 5.7 grams. The general model/formula for exponential growth of the mass is

M(t) = $\underline{M_0e^{-kt} \text{ or } M_0e^{-k(t-10)}}$.

Find a specific model for the mass in grams after t weeks. Use two significant digits for the parameter values.

$M(t) = 8.2e^{-k(t-10)}$
$M(120) = 8.2e^{-k(120-10)} = 5.7$ $k = (\ln(5.7/8.2)/-110) = 0.00330$
M(t) = $\underline{8.2e^{-0.0033(t-10)} = 8.47e^{-0.0033t}}$.

14. A farm lake is initially stocked with 14 fish. The food supply limits the population to 150. Assuming a logistic model $P(t) = \dfrac{a}{1+be^{-kt}}$ for the population, find a and b. Use two significant digits.

a = 150.

b = 9.71.

limiting value is a = 150

$\dfrac{150}{1+be^{k\cdot0}} = 14$ $\dfrac{150}{14} = 1+b$

15. A farm lake is initially stocked with a known number of fish. The food supply limits the population to known size. We partially determine a logistic model $P(t) = \dfrac{130}{1+17e^{-kt}}$ for the population. After 2 years the population is 63. Find k. Use two significant digits   k = 1.4.

$\dfrac{130}{1+17e^{-k2}} = 63$

$$\frac{130}{63} = 1 + 17e^{-k2}$$

$$\frac{130}{63} - 1 = 17e^{-k2}$$

16. A population of bacteria grows exponentially. The initial population is 500. After 4 weeks, the population is 800. Give a specific model (formula) for the population P(t) at t weeks. Calculate parameters accurate to four decimal places.

$P(t) = P_0e^{kt} = 500 \cdot e^{kt}$

$800 = 500 \cdot e^{k4}$

k4 = ln(8/5)

k = 0.1175

P(t) = $\underline{\quad 500 \cdot e^{0.1175t} \quad}$.

17. A farm lake is initially stocked with 20 fish. The food supply limits the population to 280. Assuming a logistic model $P(t) = \frac{a}{1 + be^{-kt}}$ for the population, find a and b.

$$20 = \frac{280}{1 + b \cdot 1}$$

1 + b = 280/20 = 14

a = $\underline{\quad 280 \quad}$.

b = $\underline{\quad 13 \quad}$

18. A farm lake is initially stocked with a known number of fish. The food supply limits the population to known size. We partially determine a logistic model $P(t) = \frac{120}{1 + 5e^{-kt}}$ for the population. After 2 years the population is 40. Find k. Approximate to 4 decimal places.

$$40 = \frac{120}{1 + 5e^{-k2}}$$

$$1 + 5e^{-k2} = \frac{120}{40} = 3$$

$5 \cdot e^{-k2} = 2$

$e^{-k2} = 2/5$

-k2 = ln(2/5)

k = ln(2/5)/(-2) = 0.4581

k = $\underline{\quad 0.4581 \quad}$.

19. A colony of mold grows exponentially. At 6 weeks the mass is 8.3 grams. At 9 weeks the mass is 11.5 grams. Find a specific model for the mass in grams at t weeks.

$M = 8.3e^{k(t-6)}$

$11.5 = 8.3e^{k(9-6)}$

$3k = \ln(11.5/8.3)$ $\quad k = \ln(11.5/8.3)/3 = -0.7725$

$M = 8.3e^{-0.7725(t-6)}$

20. A farm lake is initially stocked with 10 fish. The food supply limits the population to 150. Assuming a logistic model $P(t) = \frac{a}{1+be^{-kt}}$ for the population, find two of the three parameters.

$$10 = \frac{150}{1+b}$$

$1 + b = 150/10 = 15$

$b = 14$

k = ______________.

a = __150________.

b = __14_________.

21. A population of bacteria grows exponentially. After 5 weeks the population is 400. After 8 weeks, the population is 500. Give a specific model (formula) for the size of the population P(t) at t weeks. Use three significant digits for the constants in the formula.

$P(t) = 400e^{k(t-5)}$

$P(8) = 400e^{k(8-5)} = 500$

$400e^{3k} = 500$

$e^{3k} = 5/4$

$3k = \ln(5/4)$

$k = \ln(5/4)/3 = 0.07438$

P(t) = ____ $400e^{0.0744(t-5)}$ ____.

22. Values are shown for a mutual fund account from 1990 to 1996. Assume the value V(t) grows exponentially. Give a specific exponential model. Find the annual percentage growth rate for your model. Use three significant digits. Use your model to predict the account's value in 1997 to the nearest penny.

t (yrs since 1990)	**Value** (thousands of dollars)
0	10.0
1	10.7
2	11.2
3	12.2
4	12.8
5	13.9
6	15.0
7	?

V(t) = ________________. APR = __________.
Estimated V(7) = ______________________.

Showing work is unnecessary.

23. The half-life of a radioactive isotope is 90 days. If we have an initial mass of 80 grams, give the exponential model for the amount A(t) remaining after t days. Use three significant digits. Showing work is unnecessary.

A(t) = $80 \cdot 2^{-t/90}$.
ALT: $80 \cdot e^{-0.00770t}$

24. A farm lake is initially stocked with 100 fish. The food supply limits the population to 1000. Assuming a logistic model $P(t) = \frac{a}{1+be^{-kt}}$ for the population, find two of the three parameters a, b and k. Use three significant digits.

$$P(0) = 1000 = \frac{100}{1+b}$$

1 + b = 0.1

b = -0.9

a = 100.
b = -0.900.
k = ________________.

25. A bacterial culture is grown in an environment that limits its population to a known size. We know the initial population and partially determine a logistic model $P(t) = \frac{12}{1+5e^{-kt}}$ for the population measured in millions at a time t days after the culture is started. After 8 days the population is 6 million. Find k. Use three significant digits.

$$P(8) = \frac{12}{1+5e^{-k8}} = 6$$

$1 + 5e^{-8k} = 2$

$e^{-8k} = 1/5$

$-8k = \ln(0.2)$

$k = 0.201$

k = 0.201.

26. A colony of mold grows exponentially. At 3 weeks the mass is 1.7 grams. At 8 weeks the mass is 3.5 grams. Find a specific model for the mass M(t) in grams at t weeks. Use three significant digits precision for the parameters.

$M = 1.7e^{k(t-3)}$

$3.5 = 1.7e^{k(8-3)}$

$k = \ln(3.5/1.7)/5 = 0.1444$

M(t) = $1.7e^{0.1444(t-3)}$.

27. A farm lake is initially stocked with 400 fish. The food supply limits the population to 11000. Assuming a logistic model $P(t) = \frac{a}{1+be^{-kt}}$ for the population, find two of the three parameters a, b and k.

$$400 = \frac{11{,}000}{1+be^{0}}$$

$1 + b = 11{,}000/400$

$b = 110/4 - 1 = 26.5$

k = ______________.

a = 11,000.

b = 26.5.

28. A colony of mold grows exponentially. At 7 weeks the mass is 3.5 grams. At 11 weeks the mass is 6.8 grams. Find a specific model for the mass M(t) in grams at t weeks. Use three significant digits precision for the parameters.

$M = 3.5e^{k(t-7)}$

$6.8 = 3.5e^{k(11-7)}$

$k = \ln(6.8/3.5)/4 = 0.1660$

M(t) = $3.5e^{0.1660(t-7)}$.

29. (12 points) A farm lake is initially stocked with 500 fish. The food supply limits the population to 12000. Assuming a logistic model $P(t) = \frac{a}{1+be^{-kt}}$ for the population, find two of the three parameters a, b and k.

$500 = \frac{12{,}000}{1+be^{0}}$

1 + b = 12,000/500

b = 120/5 – 1 = 23

a = 12,000 .

b = 23 .

k = ______________.

30. The radioactive mass of a certain isotope decays exponentially. After 20 days the mass is 8 grams. After 60 days the mass is 2 grams. Give a specific model (formula) for the mass in grams at t days. Use four significant digits precision for the parameters in the model.

$m = 8e^{-k(t-20)}$

$2 = 8e^{-k(60-20)}$

$0.25 = e^{-k\cdot 40}$

k = -ln(0.25)/40 = 0.034657

m(t) = 0.03466 .

31. A farm lake is initially stocked with a known number of fish. The food supply limits the population to known size. We partially determine a logistic model $P(t) = \frac{14000}{1+17e^{-kt}}$ for the population. After 2 years the population is 1200. Finish specifying the logistic model. Use four significant digits precision for all parameters.

$1200 = \frac{14000}{1+17e^{-k2}}$

$1+17e^{-k2} = \frac{14000}{1200}$

$k = \ln((14000/1200-1)/17)/(-2)$ = 0.23304

P(t) = 0.2330 .

32. A farm lake is initially stocked with 80 fish. The food supply limits the population to 12,000. Assuming a logistic model $P(t) = \frac{a}{1+be^{-kt}}$ for the population, find two of the three parameters. Use four significant digits precision for your answers.

$$80 = \frac{12000}{1+b}$$

$$1+b = \frac{12000}{80} = 150$$

k = _____________________.

a = 12000.

b = 149.