MAC 2233 | Instr: Jamieson

Ch. 3 Sec. 4

Derivatives Using the Definition

1. Using the definition, $\frac{df}{dx} = \lim_{h\to 0}\left(\frac{f(x+h)-f(x)}{h}\right)$ find f'(x) if $f(x) = \frac{3}{x-5}$

2. Using the definition, $\frac{df}{dx} = \lim_{h\to 0}\left(\frac{f(x+h)-f(x)}{h}\right)$ find f'(x) if $f(x) = \frac{3}{x-5}$

3. Using the limit of the difference quotient, find f'(x) for $f(x) = \frac{7}{x-2}$ for $x \neq 2$.

4. Using the limit of the difference quotient, find f'(x) for $f(x) = \frac{9}{x-3}$ for $x \neq 3$.

5. Using the definition, $\frac{df}{dx} = \lim_{h\to 0}\left(\frac{f(x+h)-f(x)}{h}\right)$ find f'(x) for $f(x) = \frac{1}{2x-5}$.

6. Using the definition, $\frac{df}{dx} = \lim_{h\to 0}\left(\frac{f(x+h)-f(x)}{h}\right)$ find f'(x) for $f(x) = \frac{1}{4x-3}$.

7. Using the definition, $\frac{df}{dx} = \lim_{h\to 0}\left(\frac{f(x+h)-f(x)}{h}\right)$ find f'(x) for $f(x) = \frac{7}{3x-5}$.

8. Using the definition, $\frac{df}{dx} = \lim_{h\to 0}\left(\frac{f(x+h)-f(x)}{h}\right)$ find f'(x) for $f(x) = \sqrt{x-7}$.

9. Using the definition, $\frac{df}{dx} = \lim_{h\to 0}\left(\frac{f(x+h)-f(x)}{h}\right)$ find f'(x) for $f(x) = \sqrt{x+5}$.

10. Using the definition, $\frac{df}{dx} = \lim_{h\to 0}\left(\frac{f(x+h)-f(x)}{h}\right)$ find f'(x) for $f(x) = \sqrt{x-7}$.

11. Using the definition, $\frac{df}{dx} = \lim_{h\to 0}\left(\frac{f(x+h)-f(x)}{h}\right)$ find f'(x) for $f(x) = 7\sqrt{5x-3}$.

Answers:

1. $$\frac{\frac{3}{x+h-5}-\frac{3}{x-5}}{h} = \frac{3(x-5)-3(x+h-5)}{h(x+h-5)(x-5)} = \frac{-3h}{h(x+h-5)(x-5)}$$

$$= \frac{-3}{(x+h-5)(x-5)}$$

$$\lim_{h\to 0}\left(\frac{-3}{(x+h-5)(x-5)}\right) = \frac{-3}{(x-5)^2}$$

2. $$\frac{\frac{3}{x+h-5}-\frac{3}{x-5}}{h} = \frac{3(x-5)-3(x+h-5)}{h(x+h-5)(x-5)} = \frac{-3h}{h(x+h-5)(x-5)} = \frac{-3}{(x+h-5)(x-5)}$$

Derivatives Using the Definition

$$\lim_{h\to 0}\left(\frac{-3}{(x+h-5)(x-5)}\right)=\frac{-3}{(x-5)^2}$$

3. $$\frac{f(x+h)-f(x)}{h}=\frac{\frac{7}{x+h-2}-\frac{7}{x-2}}{h}=\frac{7(x-2)-7(x+h-2)}{h(x-2)(x+h-2)}=\frac{-7h}{h(x-2)(x+h-2)}$$

$$=\frac{-7}{(x-2)(x+h-2)}$$

$$\lim_{h\to 0}\left(\frac{-7}{(x-2)(x+h-2)}\right)=\frac{-7}{(x-2)^2}$$

4. $$\frac{f(x+h)-f(x)}{h}=\frac{\frac{9}{x+h-3}-\frac{9}{x-3}}{h}=\frac{9(x-3)-9(x+h-3)}{h(x-3)(x+h-3)}=\frac{-9h}{h(x-3)(x+h-3)}$$

$$=\frac{-9}{(x-3)(x+h-3)}$$

$$\lim_{h\to 0}\left(\frac{-9}{(x-3)(x+h-3)}\right)=\frac{-9}{(x-3)^2}$$

5. $$\frac{\frac{1}{2(x+h)-5}-\frac{1}{2x-5}}{h}=\frac{2x-5-(2(x+h)-5)}{h(2(x+h)-5)(2x-5)}=\frac{-2h}{h(2(x+h)-5)(2x-5)}=\frac{-2}{(2(x+h)-5)(2x-5)}$$

$$\lim_{h\to 0}\left(\frac{-2}{(2(x+h)-5)(2x-5)}\right)=\frac{-2}{(2x-5)^2}$$

6. $$\frac{\frac{1}{4(x+h)-3}-\frac{1}{4x-3}}{h}=\frac{4x-3-(4(x+h)-3)}{h(4(x+h)-3)(4x-3)}=\frac{-4h}{h(4(x+h)-3)(4x-3)}=\frac{-4}{(4(x+h)-3)(4x-3)}$$

$$\lim_{h\to 0}\left(\frac{-4}{(4(x+h)-3)(4x-3)}\right)=\frac{-4}{(4x-3)^2}$$

7. $$\frac{\frac{7}{3(x+h)-5}-\frac{7}{3x-5}}{h}=\frac{7(3x-5)-7(3x+3h-5)}{h(3x+3h-5)(3x-5)}=\frac{-7\cdot 3h}{h(3x+3h-5)(3x-5)}=\frac{-7\cdot 3}{(3x+3h-5)(3x-5)}$$

$$\lim_{h\to 0}\left(\frac{-7\cdot 3}{(3x+3h-5)(3x-5)}\right)=\frac{-7\cdot 3}{(3x-5)^2}$$

8. $$\lim_{h\to 0}\left(\frac{-4}{(4(x+h)-3)(4x-3)}\right)=\frac{-4}{(4x-3)^2}$$

Derivatives Using the Definition

9. $\dfrac{\sqrt{x+h-7}-\sqrt{x-7}}{h}=\dfrac{\sqrt{x+h-7}-\sqrt{x-7}}{h}\cdot\dfrac{\sqrt{x+h-7}+\sqrt{x-7}}{\sqrt{x+h-7}+\sqrt{x-7}}=\dfrac{x+h-7-(x-7)}{h(\sqrt{x+h-7}+\sqrt{x-7})}$

$=\dfrac{h}{h(\sqrt{x+h-7}+\sqrt{x-7})}=\dfrac{1}{\sqrt{x+h-7}+\sqrt{x-7})}$

$\lim\limits_{h\to 0}\left(\dfrac{1}{\sqrt{x+h-7}+\sqrt{x-7})}\right)=\dfrac{1}{2\sqrt{x-7}}$

10. $\dfrac{\sqrt{x+h+5}-\sqrt{x+5}}{h}\cdot\dfrac{\sqrt{x+h+5}+\sqrt{x+5}}{\sqrt{x+h+5}+\sqrt{x+5}}=\dfrac{(x+h+5)-(x+5)}{h(\sqrt{x+h+5}+\sqrt{x+5})}$

$=\dfrac{h}{h(\sqrt{x+h+5}+\sqrt{x+5})}$

$\lim\limits_{h\to 0}\left(\dfrac{1}{\sqrt{x+h+5}+\sqrt{x+5}}\right)=\dfrac{1}{2\sqrt{x+5}}$

11. $\dfrac{\sqrt{x+h-7}-\sqrt{x-7}}{h}=\dfrac{\sqrt{x+h-7}-\sqrt{x-7}}{h}\cdot\dfrac{\sqrt{x+h-7}+\sqrt{x-7}}{\sqrt{x+h-7}+\sqrt{x-7}}=\dfrac{x+h-7-(x-7)}{h(\sqrt{x+h-7}+\sqrt{x-7})}$

$=\dfrac{h}{h(\sqrt{x+h-7}+\sqrt{x-7})}$

$\lim\limits_{h\to 0}\left(\dfrac{1}{\sqrt{x+h-7}+\sqrt{x-7}}\right)=\dfrac{1}{2\sqrt{x-7}}$

12. $\dfrac{7\sqrt{5(x+h)-3}-7\sqrt{5x-3}}{h}=\dfrac{7\sqrt{5(x+h)-3}-7\sqrt{5x-3}}{h}\cdot\dfrac{7\sqrt{5x+5h-3}+7\sqrt{5x-3}}{7\sqrt{5x+5h-3}+7\sqrt{5x-3}}$

$=\dfrac{7^2(5x+5h-3)-7^2(5x-3)}{h(7\sqrt{5x+5h-3}+7\sqrt{5x-3})}=\dfrac{7^2\cdot 5h}{h(7\sqrt{5x+5h-3}+7\sqrt{5x-3})}=\dfrac{7^2\cdot 5}{7\sqrt{5x+5h-3}+7\sqrt{5x-3}}$

$\lim\limits_{h\to 0}\left(\dfrac{7^2\cdot 5}{7\sqrt{5x+5h-3}+7\sqrt{5x-3}}\right)=\dfrac{7^2\cdot 5}{2\sqrt{x-7}}$