

# Product and Quotient Rules with Chain Rule

## Ch 4 Sec 3 – 4

Differentiate:

1.  $x^5 e^{7x}$

2.  $(x+30)^{27} (x-15)^{92}$

3.  $(x+20)^{15} \ln(x-3)$

4.  $(7x^4 + 1)^5 e^{7x}$

5.  $e^{7x} \ln(5x^2 + 3)$

6.  $x^5 \ln(x+1) e^{7x}$

7.  $(x^2 + 1)^7 (3x^5 + 1)^9$

8.  $e^{2x} \ln(x-5)$

9.  $(x+3)^9 e^{3x}$

10.  $(x+7)^7 \ln(x+3)$

11.  $(x+1)^9 (x+2)^{10}$

12.  $(2x+1)^7 (5x^2 + 1)^9$

13.  $7x^5 e^{3x}$

14.  $900(x+0.1)^{0.4} (x+2)^{0.6}$

15.  $e^{3x} 10^x 2^x$

16.  $\frac{1}{x^2 + 9x - 3}$

17.  $2 \cdot \frac{5}{e^x - 7}$

18.  $\frac{140}{1 + 6e^{-0.02x}}$

19.  $(\ln(2^x + 1))^9$

20.  $(\log(2^x + 1))^9$

21.  $e^{(\ln(x)+1)^5}$

22.  $f(x) = (x^2 + 5)^{10}$

23.  $f(x) = (2^x + 1)^7$

24.  $f(x) = (x^3 + 5)^{10}$

25.  $f(x) = (2^x + 1)^6$

26.  $f(x) = (4x^2 + 5x + 3)^{-2}$

27.  $f(x) = (\sqrt{x} - 2x)^4$

28.  $f(x) = e^{5x^2}$

29.  $f(x) = (x^3 + 1)^8$

30.  $f(x) = \ln((x+2)^4)(x+1)^5$

31.  $f(x) = \ln((x+2)^4) e^{3x}$

32.  $f(x) = \frac{120}{1 + 7e^{-0.03x}}$

33.  $f(x) = (x^2 + 5)^{10}$

34.  $f(x) = (1.3^x + 5)^7$

35.  $f(x) = (3x^4 + 5)^7$

36.  $f(x) = e^{(3x^5)}$

37.  $f(x) = 10^{(3x^5)}$

38.  $f(x) = \frac{1300}{1 + 25e^{-0.032t}}$

39.  $f(x) = (3e^x + 7)^9$

40.  $f(x) = (3\ln(x) + 7)^3$

41.  $f(x) = \ln(3x^7 + 1)$

42.  $f(x) = \ln\left(e^{(x^2)} + 1\right)$

43.  $f(x) = \ln((3x+1)^5 + 1)$

44.  $f(x) = \frac{x^5 \ln(x+1)}{e^x}$

45.  $f(x) = x^2 e^{3x}$

46.  $f(x) = (x+1)^9 (x-3)^{17}$

47.  $f(x) = (11x-1)^5 (3x+2)^9$

48.  $f(x) = e^{3x} \ln(x)$

49.  $f(x) = \ln(x+1) \ln(x+2)$

50.  $f(x) = x^8 \ln(x+1)$

51.  $f(x) = x^9 \cdot e^{2x} \cdot \ln(x-7)$

52.  $f(x) = (x+1)^9 (x+2)^{15} (x+3)^{12}$

53.  $f(x) = \ln(2x-7) \ln(5x-7) \ln(x)$

54.  $f(x) = e^{3x} \ln(x)$

55.  $f(x) = \ln(2x+1) \ln(x+5)$

56.  $f(x) = 5x^9 \ln(x+1)$

57.  $f(x) = x^7 e^{5x} \ln(x+3)$

58.  $f(x) = (x+1)^9 (2x+5)^{17} (x-12)^{95}$

59.  $f(x) = e^{x^2} 10^x$

60.  $f(x) = \ln(x-7) \ln(3x+2) \ln(5x-3)$

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61.  $f(x) = (5e^{3x^2} + 1)^9$

62.  $x^9 \ln(x^5 + 1)$

63.  $\ln((x + 1)^5 e^{3x})$

64.  $\ln((x + 1)^5 (x + 2)^4)$

65.  $\ln((x + 2)^4 (x + 1)^5)$

66.  $\ln((x + 2)^4 e^{3x})$

67.  $\ln((x + 2)^4 e^{5x})$

68.  $\ln(x^5 (e^{3x} + 1))$

69.  $\ln(x^5 + 1)e^{3x}$

70.  $(7x^2 + 1)^9 (2x + 3)^{15}$

71.  $(x + 1)^7 \ln(x - 3)e^{7x}$

72.  $\ln((x^2 + 7)e^x)$

73.  $\ln(x^2 + 7)e^x$

74.  $2(\ln(x^{12} + 3) + 1)^7$

75.  $\frac{e^{7x}}{3x^5}$

76.  $\frac{40}{1 + 3e^{-0.7t}}$

77.  $\ln((x + 1)^4 e^x)$

78.  $\ln((x + 1)^4)e^x$

79.  $f(x) = \ln((x + 2)^4 e^{5x})$

80.  $f(x) = \ln(x^5 (e^{3x} + 1))$

81.  $f(x) = \ln(x^5 + 1)e^{3x}$

82.  $f(x) = (x^2 + 5)^{10}$

83.  $f(x) = (x^3 + 5)^{10}$

84.  $f(x) = x^9 \ln(x^5 + 1)$

85.  $f(x) = \ln((x + 1)^5 + 7)$

86.  $f(x) = \ln((x + 1)^5 e^{3x})$

87.  $f(x) = \ln((x + 1)^5 (x + 2)^4)$

88.  $f(x) = \ln(x^6 e^{2x})$

89.  $f(x) = e^{x \ln(x + 1)}$

90.  $f(x) = \ln(x^6)e^{2x}$

91.  $f(x) = \ln(1 + e^{x^5})$

92.  $f(x) = \ln(1 + e^x)x^5$

93. The percentage of children living with their grandparents between 1970 and 2000 can be modeled by the equation  $p(t) = [3 + 0.216e^{0.09263t}]$  percent  $t$  years after 1970. Write a rate of change formula for  $p$ . How rapidly was the percentage of children living with their grandparents growing in 1995?

94. Imagine that you invest \$1500 in a savings account at 4% annual interest compounded continuously. Write an equation for the balance in the account after  $t$  years. Write an equation for the rate of change of the balance. What is the rate of change of the balance at the end of 1 year? 2 years?

95. In October of 1999, iGo Corp. offered 5 million shares of public stock at \$9 per share. Revenue for the two years preceding the stock offering can be modeled by the equation  $R(q) = 2.9 + 0.0314e^{0.622285q}$  million dollars  $q$  quarters after the beginning of 1998. Write the rate of change formula of  $R$ . Calculate revenue and rate of change of revenue for June 1998, June 1999, June 2000.

96. Dispatchers at a sheriff's office record the total number of calls received since 5am in 3-hour intervals. Total calls for a typical day can be modeled as  $N(t) = \frac{1342}{1 + 36.80e^{-0.2589t}}$   $t$  hours after 5am. Find the rate of change formula for the model.

97. A model for the number of states associated with the national P.T.A. organization is  $m(x) = \frac{49}{1 + 36.0660e^{-0.206743x}}$  states  $x$  years after 1895. Write the derivative of  $m$ . How many states had national P.T.A. membership in 1902? How rapidly was the number of states in the P.T.A. growing in 1890? 1915? 1927?

98. Civilian deaths due to the influenza epidemic in 1918 can be modeled as

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$c(t) = \frac{93,700}{1 + 5095.9634e^{-1.097175t}}$  deaths  $t$  weeks after August 31, 1918. How rapidly was the number of deaths growing on September 28, 1918?

**Answers:**

1.  $x^5 e^{7x} 7 + 5x^4 e^{7x}$

2.  $(x + 30)^{27} 92(x - 15)^{91} + 27(x + 30)^{26} (x - 15)^{92}$

3.  $(x + 20)^{15} \frac{1}{x - 3} + 15(x + 20)^{14} \ln(x - 3)$

4.  $(7x^4 + 1)^5 e^{7x} 7 + 5(7x^4 + 1)^4 28x^3 e^{7x}$

5.  $e^{7x} \cdot \frac{1}{5x^2 + 3} \cdot 10x + e^{7x} 7 \ln(5x^2 + 3)$

6.  $5x^4 \ln(x + 1)e^{7x} + x^5 \frac{1}{x + 1} e^{7x} + x^5 \ln(x + 1)e^{7x} 7$

7.  $(x^2 + 1)^7 9(3x^5 + 1)^8 15x^4 + 7(x^2 + 1)^6 2x(3x^5 + 1)^9$

8.  $e^{2x} \frac{1}{x - 5} + e^{2x} 2 \ln(x - 5)$

9.  $(x + 3)^9 e^{3x} 3 + 9(x + 3)^8 e^{3x}$

10.  $(x + 7)^7 \frac{1}{x + 3} + 7(x + 7)^6 \ln(x + 3)$

11.  $(x + 1)^9 10(x + 2)^9 + 9(x + 1)^8 (x + 2)^{10}$

12.  $(2x + 1)^7 9(5x^2 + 1)^8 10x + 7(2x + 1)^6 2(5x^2 + 1)^9$

13.  $7[x^5 e^{3x} 3 + 5x^4 e^{3x}]$

14.  $900[(x + 0.1)^{0.4} 0.6(x + 2)^{-0.4} + 0.4(x + 0.1)^{-0.6}(x + 2)^{0.6}]$

15.  $e^{3x} 3 \cdot 10^x 2^x + e^{3x} \ln(10) 10^x 2^x + e^{3x} 10^x \ln(2) 2^x$

16.  $-(x^2 + 9x - 3)^{-2} (2x + 9)$

17.  $-10(e^x - 7)^{-2} e^x$

18.  $-140(1 + 6e^{-0.02x})^{-2}$

$\cdot 6e^{-0.02x}(-0.02)$

19.  $9(\ln(2^x + 1))^8 \cdot \frac{1}{2^x + 1} \cdot 2^x \ln(2)$

20.  $9(\log(2^x + 1))^8 \cdot \frac{1}{\ln(10)(2^x + 1)} \cdot 2^x \ln(2)$

21.  $e^{(\ln(x)+1)^5} \cdot 5(\ln(x)+1)^4 \cdot \frac{1}{x}$

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22.  $f(x) = 10(x^2 + 5)^9 2x$

23.  $f(x) = 7(2^x + 1)^6 2^x \ln(2)$

24.  $f(x) = 10(x^3 + 5)^9 3x^2$

25.  $f(x) = 6(2^x + 1)^5 2^x \ln(2)$

26.  $-2(4x^2 + 5x + 3)^{-3}(8x + 5)$

27.  $f(x) = (\sqrt{x} - 2x)^4 = (x^{1/2} - 2x)^4$

$$f'(x) = 4(x^{1/2} - 2x)^3 \left( \frac{1}{2}x^{-1/2} - 2 \right)$$

28.  $f'(x) = 10xe^{5x^2}$

29.  $f(x) = 8(x^3 + 1)^7 \cdot 3x^2$

30.  $10(x^2 + 5)^9 2x$

31.  $f(x) = 10(x^3 + 5)^9 \cdot 3x$

32.  $f(x) = 120(1 + 7e^{-0.03x})^{-1}$

$$f'(x) = -120(1 + 7e^{-0.03x})^{-2} \cdot 7 \cdot e^{-0.03x}(-0.03)$$

33.  $f(x) = 10(x^2 + 5)^9 \cdot 2x$

34.  $f(x) = 7(1.3^x + 5)^6 \cdot \ln(1.3) \cdot 1.3^x$

35.  $7 \cdot (3x^4 + 5)^6 \cdot 12x^3$

36.  $e^{(3x^5)} \cdot 15x^4$

37.  $\ln(10) \cdot 10^{(3x^5)} \cdot 15x^4$

38. 
$$\frac{\left(1 + 25e^{-0.032t}\right) \cdot 0 - 1300 \cdot \left(-0.032 \cdot 25e^{-0.032t}\right)}{\left(1 + 25e^{-0.032t}\right)^2}$$

$$= \frac{-1300 \cdot \left(-0.032 \cdot 25e^{-0.032t}\right)}{\left(1 + 25e^{-0.032t}\right)^2}$$

$$= \frac{-1300 \cdot \left(-0.032 \cdot 25e^{-0.032t}\right)}{\left(1 + 25e^{-0.032t}\right)^2}$$

$$= \frac{-1300 \cdot \left(-0.032 \cdot 25e^{-0.032t}\right)}{\left(1 + 25e^{-0.032t}\right)^2}$$

39.  $9(3e^x + 7)^8 \cdot 3e^x$

40.  $3 \cdot (3\ln(x) + 7)^2 \cdot \frac{3}{x}$

41.  $\frac{1}{3x^7 + 1} \cdot 21x^6$

42.  $\frac{1}{e^{(x^2)} + 1} \cdot e^{(x^2)} \cdot 2x$

43.  $\frac{1}{((3x + 1)^5 + 1)} \cdot 5(3x + 1)^4 \cdot 3$

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$$44. f(x) = \frac{x^5 \ln(x+1)}{e^x} = x^5 \ln(x+1)e^{-x}$$

$$f'(x) = 5x^4 \ln(x+1)e^{-x} + x^5 \frac{1}{x+1} e^{-x} + x^5 \ln(x+1)e^{-x} \cdot (-1)$$

$$45. x^2 \cdot e^{3x} \cdot 3 + 2x \cdot e^{3x}$$

$$46. (x+1)^9 17(x-3)^{16} + 9(x+1)^8(x-3)^{17}$$

$$47. (11x-1)^5 9(3x+2)^9 \cdot 3 + 5(11x-1)^5 \cdot 11(3x+2)^9$$

$$48. e^{3x} \frac{1}{x} + e^{3x} \cdot 3 \cdot \ln(x)$$

$$49. \ln(x+1) \frac{1}{x+2} + \frac{1}{x+1} \ln(x+2)$$

$$50. x^8 \frac{1}{x+1} + 8x^7 \ln(x+1)$$

$$51. 9x^8 \cdot e^{2x} \cdot \ln(x-7) + x^9 \cdot e^{2x} \cdot 2 \cdot \ln(x-7) + x^9 \cdot e^{2x} \cdot \frac{1}{x-7}$$

$$52. 9(x+1)^8(x+2)^{15}(x+3)^{12}$$

$$+ (x+1)^9 15(x+2)^{14}(x+3)^{12}$$

$$+ (x+1)^9(x+2)^{15} 12(x+3)^{11}$$

$$53. \frac{1}{2x-7} \cdot 2 \cdot \ln(5x-7) \ln(x) + \ln(2x-7) \frac{1}{5x-7} \cdot 5 \cdot \ln(x) + \ln(2x-7) \ln(5x-7) \frac{1}{x}$$

$$54. e^{3x} \frac{1}{x} + e^{3x} \cdot 3 \ln(x)$$

$$55. \ln(2x+1) \frac{1}{x+5} + \frac{1}{2x+1} \cdot 2 \ln(x+5)$$

$$56. f(x) = 5x^9 \ln(x+1) \cdot 5 \left[ x^9 \frac{1}{x+1} + 9x^8 \ln(x+1) \right]$$

$$57. 7x^6 e^{5x} \ln(x+3) + x^7 \cdot 5e^{5x} \ln(x+3) + x^7 e^{5x} \frac{1}{x+3}$$

$$58. 9(x+1)^8(2x+5)^{17}(x-12)^{95}$$

$$+ (x+1)^9 \cdot 17 \cdot 2(2x+5)^{16}(x-12)^{95}$$

$$+ (x+1)^9(2x+5)^{17} \cdot 95(x-12)^{94}$$

$$59. e^{2x} 10^x + e^x \cdot \ln(2) 2^x 10^x + e^{2x} \cdot \ln(10) 10^x$$

$$60. \frac{1}{x-7} \ln(3x+2) \ln(5x-3) + \ln(x-7) \frac{1}{3x+2} \cdot 3 \ln(5x-3) + \ln(x-7) \ln(3x+2) \frac{1}{5x-3} \cdot 5$$

$$61. 9 \left( 5e^{3x^2} + 1 \right)^8 \cdot 5e^{3x^2} \cdot 6x$$

$$62. f'(x) = x^9 \frac{1}{x^5+1} 5x^4 + 9x^8 \ln(x^5+1)$$

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$$63. f'(x) = \frac{1}{(x+1)^5 e^{3x}} \cdot \left[ (x+1)^5 e^{3x} 3 + 5(x+1)^4 e^{3x} \right]$$

$$64. f'(x) = \frac{1}{(x+1)^5 (x+2)^4} \cdot \left[ (x+1)^5 4(x+2)^3 + 5(x+1)^4 (x+2)^4 \right]$$

$$65. f'(x) = \ln((x+2)^4) \cdot 5(x+1)^4 + \frac{1}{(x+2)^4} 4(x+2)^3 \cdot (x+1)^5$$

$$66. f'(x) = \ln((x+2)^4) \cdot e^{3x} 3 + \frac{1}{(x+2)^4} 4(x+2)^3 \cdot e^{3x}$$

$$67. f'(x) = \frac{1}{(x+2)^4 e^{5x}} \cdot \left[ (x+2)^4 e^{5x} 5 + 4(x+2)^3 e^{5x} \right]$$

$$68. f'(x) = \frac{1}{x^5 (e^{3x} + 1)} \left[ x^5 e^{3x} 3 + 5x^4 (e^{3x} + 1) \right]$$

$$69. f'(x) = \ln(x^5 + 1) e^{3x} 3 + \frac{1}{x^5 + 1} 5x^4 e^{3x}$$

$$70. (7x^2 + 1)^9 15(2x+3)^{14} 2 + 9(7x^2 + 1)^8 14x(2x+3)^{15}$$

$$71. 7(x+1)^6 \ln(x-3) e^{7x} + (x+1)^7 \frac{1}{x-3} e^{7x} + (x+1)^7 \ln(x-3) e^{7x} 7$$

$$72. \frac{1}{(x^2 + 7)e^x} \left[ (x^2 + 7)e^x + 2xe^x \right]$$

$$73. \ln(x^2 + 7) e^x + \frac{1}{x^2 + 7} 2xe^x$$

$$74. 14(\ln(x^{12} + 3) + 1)^6 \frac{1}{x^{12} + 3} 12x^{11}$$

$$75. \frac{3x^5 e^{7x} 7 - e^{7x} 15x^4}{9x^{10}}$$

$$76. \frac{(1 + 3e^{-0.7t}) \cdot 0 - 40 \cdot 3e^{-0.7t} (-0.7)}{(1 + 3e^{-0.7t})^2}$$

$$77. \frac{1}{(x+1)^4 e^x} \left[ (x+1)^4 e^x + 4(x+1)^3 e^x \right]$$

$$78. \ln((x+1)^4) e^x + \frac{1}{(x+1)^4} 4(x+1)^3 e^x$$

$$79. \ln((x+2)^4) 5(x+1)^4 + \frac{1}{(x+2)^4} 4(x+2)^3 (x+1)^5$$

$$80. \ln((x+2)^4) e^{3x} 3 + \frac{1}{(x+2)^4} 4(x+2)^3 e^{3x}$$

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$$81. \frac{1}{(x+2)^4 e^{5x}} \left[ (x+2)^4 e^{5x} 5 + 4(x+2)^3 e^{5x} \right]$$

$$82. \frac{1}{x^5 (e^{3x} + 1)} \left[ x^5 e^{3x} 3 + 5x^4 (e^{3x} + 1) \right]$$

$$83. \ln(x^5 + 1) e^{3x} 3 + \frac{1}{x^5 + 1} 5x^4 e^{3x}$$

$$84. \frac{x^9}{x^5 + 1} * 5x^4 + 9x^8 \ln(x^5 + 1)$$

$$85. \frac{1}{(x+1)^5 + 7} * 5(x+1)^4$$

$$86. \frac{1}{(x+1)^5 e^{3x}} \left[ (x+1)^5 e^{3x} * 3 + 5(x+1)^4 e^{3x} \right]$$

$$87. \frac{1}{(x+1)^5 (x+2)^4} \left[ (x+1)^5 4(x+2)^3 + 5(x+1)^4 (x+2)^4 \right]$$

$$88. \frac{1}{x^6 e^{2x}} \left[ x^6 e^{2x} \cdot 2 + 6x^5 e^{2x} \right]$$

$$89. e^{x \ln(x+1)} \left[ x \cdot \frac{1}{x+1} + \ln(x+1) \right]$$

$$90. \ln(x^6) e^{2x} \cdot 2 + \frac{1}{x^6} \cdot x^5 e^{2x}$$

$$91. \frac{1}{1 + e^x x^5} \left[ e^x 5x^4 + e^x x^5 \right]$$

$$92. \ln(1 + e^x) 5x^4 + \frac{1}{1 + e^x} e^x \cdot x^5$$

$$93. p'(t) = 0.216 * 0.09263 * e^{0.09263t}$$

$$p'(25) = 0.216 * 0.09263 * e^{0.09263 * 25} = 0.20273226 \text{ percent per year}$$

$$94. A(t) = 1500 e^{0.04t} \quad A'(t) = 1500 * 0.04 * e^{0.04t}$$

$$A'(1) = 62.44 \text{ dollars per year} \quad A'(2) = 64.99 \text{ dollars per year}$$

$$95. R'(q) = 0.0314 * 0.622285 e^{0.622285q}$$

Q	R(q)	R'(q)
2	3.009003	0.0678311
6	4.213588	0.817426
10	18.72993	9.850729

$$96. N'(t) = \frac{-1342 * 36.80 e^{-0.2589t} (-0.2589)}{(1 + 36.80 e^{-0.2589t})^2}$$

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$$97. m'(x) = \frac{-49 * 36.0660e^{-0.206743x}(-0.206743)}{(1 + 36.0660e^{-0.206743x})^2}$$

$$m(7) = \frac{49}{1 + 36.0660e^{-0.206743*7}} = 5$$

$$m'(x) = \frac{-49 * 36.0660e^{-0.206743*x}(-0.206743)}{(1 + 36.0660e^{-0.206743*x})^2}$$

x	m'(x)
-5	0.098
20	2.35
32	0.44

$$98. c'(t) = \frac{-93,700 * 5095.9634e^{-1.097175t}(-1.097175)}{(1 + 5095.9634e^{-1.097175t})^2}$$

$$c'(4) = \frac{-93,700 * 5095.9634e^{-1.097175*4}(-1.097175)}{(1 + 5095.9634e^{-1.097175*4})^2} = 1574.55$$