

Extrema, Inflection Points and Applications

Ch 5. Sec 1 – 3

1. .

$$f(x) = (x - 50)^{4/5}(x + 100)^{3/5} \quad f'(x) \approx \frac{1.4(x + 35.7)}{(x - 50)^{1/5}(x + 100)^{2/5}} \quad f''(x) \approx \frac{0.56(x + 153)(x - 81.7)}{(x - 50)^{6/5}(x + 100)^{7/5}}$$

Rel min at x = _____

Concave up: _____

Rel max at x = _____

Infl points at x = _____

Other critical points at x = _____

Possible infl points at x = _____

f	+	-100	+	-35.7	-	50	+
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f'	-	-153	+	-100	-	50	-	81.7	+
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2. .

$$f(x) = (x - 50)^{2/5}(x + 100)^{1/5} \quad f'(x) = \frac{0.6(x + 50)}{(x - 50)^{3/5}(x + 100)^{4/5}} \quad f''(x) = \frac{0.24(x^2 + 100x + 15000)}{(x - 50)^{8/5}(x + 100)^{9/5}}$$

(f''(x) has no zeroes.)

f	+	-100	+	-50	-	50	+
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f'	-	-100	+	50	+
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Rel min at x = _____

Rel max at x = _____

Other critical points at x = _____

Concave up: _____

Infl points at x = _____

Possible infl points at x = _____

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3. $f(x) = (x - 50)^2(x + 100)^{2/3}$ $f'(x) \approx \frac{3.667(x + 72.7)}{(x - 50)^2(x + 100)^{1/3}}$ $f''(x) \approx \frac{9.778(x + 108)(x + 37.2)}{(x - 50)(x + 100)^{4/3}}$

f' + -100 - -72.7 + 50 +

f'' - -108 + -100 + -37.2 - 50 +

Rel min at $x =$ _____

Rel max at $x =$ _____

Other critical points at $x =$ _____

Concave up: _____

Infl points at $x =$ _____

Possible infl points at $x =$ _____

4. Identify and classify the critical points and possible inflection points for $f(x) = x^5(x - 90)^8$ given

$f(x) = 13x^4(x - 90)^7(x - 34.6)$ and $f'(x) = 156x^3(x - 90)^6(x - 22.0)(x - 47.3)$

f' + 0 + 34.6 - 90 +

f'' - 0 + 22.0 - 47.3 + 90 +

point of dim returns at $x =$ _____

rel max at $x =$ _____

other infl points at $x =$ _____

rel min at $x =$ _____

other possible infl points at $x =$ _____

other critical points at $x =$ _____

In #5 – 14, identify and classify critical points and possible inflection points.

5. $f(x) = 16x^{3/4}(x - 10)$ $f'(x) \approx \frac{28(x - 4.29)}{x^{1/4}}$ $f''(x) \approx \frac{21(x + 1.43)}{x^{5/4}}$

f' + 0 - 4.29 +

f'' + -1.43 - 0 +

rel max at $x =$ _____

point of dim returns at $x =$ _____

rel min at $x =$ _____

other infl points at $x =$ _____

other critical points at $x =$ _____

other possible infl points at $x =$ _____

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$$9. f(x) = 25x^4(x-10)^{1/5} \quad f'(x) = \frac{105x^3(x-9.52)}{(x-10)^{4/5}} \quad f''(x) = \frac{336x^2(x-8.33)(x-10.71)}{(x-10)^{9/5}}$$

$$f' \quad \begin{array}{ccccccc} & + & 0 & & - & & 9.52 & + & 10 & + \\ \hline & & | & & & & | & & | & \end{array}$$

$$f'' \quad \begin{array}{ccccccc} & - & 0 & & - & & 8.33 & + & 10 & - & 10.71 & + \\ \hline & & | & & & & | & & | & & | & \end{array}$$

rel max at x = _____

rel min at x = _____

other critical points at x = _____

point of dim returns at x = _____

other infl points at x = _____

other possible infl points at x = _____

$$10. f(x) = -x^5(x-90)^8 \quad f'(x) = -13x^4(x-90)^7(x-34.6) \quad f''(x) = -156x^3(x-90)^6(x-22.0)(x-47.3)$$

$$f' \quad \begin{array}{ccccccc} & - & 0 & & - & & 34.6 & + & 90 & - \\ \hline & & | & & & & | & & | & \end{array}$$

$$f'' \quad \begin{array}{ccccccc} & + & 0 & - & 22.0 & + & 47.3 & - & 90 & - \\ \hline & & | & & | & & | & & | & \end{array}$$

rel max at x = _____

rel min at x = _____

other crit points: x _____

point of dim returns: x = _____

other infl points: x = _____

other possible infl points: x = _____

$$11. f(x) = -25x^{2/5}(x-4)^{1/5}$$

$$f'(x) = -5 \frac{3x-8}{x^{3/5}(x-4)^{4/5}} \approx \frac{-15(x-2.67)}{x^{3/5}(x-4)^{4/5}} \quad \text{and} \quad f''(x) = \frac{2(3x^2-16x+48)}{x^{8/5}(x-4)^{9/5}}$$

$$f' \quad \begin{array}{ccccccc} & - & 0 & + & 2.67 & - & 4 & - \\ \hline & & | & & | & & | & \end{array}$$

$$f'' \quad \begin{array}{ccccccc} & - & 0 & & - & & 4 & + \\ \hline & & | & & & & | & \end{array}$$

rel max: x = _____

rel min: x = _____

other crit points: x = _____

point of dim returns: x = _____

other infl points: x = _____

other possible infl points: x = _____

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$$12. f(x) = x^2(x-20)^3 \quad f'(x) = 5x(x-20)^2(x-8) \quad f''(x) = 20(x-20)(x^2 - 16x + 40)$$

$$f' \quad \begin{array}{ccccccc} & + & & - & & + & \\ & & | & & | & & | \\ & & 0 & & 8 & & 20 & + \end{array}$$

$$f'' \quad \begin{array}{ccccccc} & & & - & & & + \\ & & & & & & | \\ & & & & & & 20 & + \end{array}$$

point of dim returns at $x = \underline{\hspace{2cm}}$.rel max at $x = \underline{\hspace{2cm}}$.other infl points at $x = \underline{\hspace{2cm}}$.rel min at $x = \underline{\hspace{2cm}}$.other possible infl points at $x = \underline{\hspace{2cm}}$.other critical points at $x = \underline{\hspace{2cm}}$.

$$13. f(x) = 25x^4(x-10)^{1/5} \quad f'(x) = \frac{105x^3(x-9.52)}{(x-10)^{4/5}} \quad f''(x) = \frac{336x^2(x-8.33)(x-10.71)}{(x-10)^{9/5}}$$

$$f' \quad \begin{array}{ccccccc} & + & & - & & + & \\ & & | & & | & & | \\ & & 0 & & 9.52 & & 10 & + \end{array}$$

$$f'' \quad \begin{array}{ccccccc} & - & & - & & + & \\ & & | & & | & & | \\ & & 0 & & 8.33 & & 10 & - & 10.71 & + \end{array}$$

point of dim returns at $x = \underline{\hspace{2cm}}$.rel max at $x = \underline{\hspace{2cm}}$.other infl points at $x = \underline{\hspace{2cm}}$.rel min at $x = \underline{\hspace{2cm}}$.other possible infl points at $x = \underline{\hspace{2cm}}$.other critical points at $x = \underline{\hspace{2cm}}$.

14. The median size a new single family house built in the United States between 1897 and 2001 can be modeled by $H(x) = 0.4x^3 - 15x^2 + 221x + 826$ sq ft, x years after 1980.

a. Use calculus to find the inflection point(s) for $H(x)$

b. For each point you identify in (a.), indicate whether H is increasing most rapidly, least rapidly, or neither at that point.

15. The percentage of southern Australian grasshopper eggs that hatch as a function of temperature (for temperatures between 5°C and 20°C) is modeled by $P(t) = 0.007t^4 + 0.5t^3 - 13t^2 + 137t - 405$ percent, where t is the temperature in $^\circ\text{C}$.

a. Use calculus to find the inflection point(s) for $P(t)$

b. For each point you identify in (a.), indicate whether P is increasing most rapidly, least rapidly, or neither at that point.

16. Find and classify extrema for $f(x) = 0.01x^4 + 400x - 100$ for all x .

17. Find and classify extrema for $f(x) = 0.01x^3 - 4x + 100$ for all x .

18. Find and classify extrema for $f(x) = 200x(0.7^x) + 50$ for $x \geq 0$.

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19. The amount of refuse (in millions of tons) taken to a landfill outside a city is about

$$A(t) = 0.01t^3 - 0.5t^2 + 10t + 40, \text{ million tons } t \text{ years after 1970.}$$

Use calculus to find any inflection points. Include units in your answer.

Answers:

1. Rel min at $x = \underline{50}$.

Rel max at $x = \underline{-35.7}$.

Other critical points at $x = \underline{-100}$.

Concave up: $\underline{(-153, -100), (81.7, \infty)}$.

Infl points at $x = \underline{-153, -100, 81.7}$.

Possible infl points at $x = \underline{50}$.

2.

Rel min at $x = \underline{50}$. Rel max at $x = \underline{-50}$.

Other critical points at $x = \underline{-100}$.

Concave up: $\underline{(-100, 50), (50, \infty)}$.

Infl points at $x = \underline{-100}$. Possible infl points at $x = \underline{50}$.

3.

Rel min at $x = \underline{-72.7}$. Rel max at $x = \underline{-100}$.

Other critical points at $x = \underline{50}$.

Concave up: $\underline{(-108, -100), (-100, -37.2), (50, \infty)}$.

Infl points at $x = \underline{-108, -37.2, 50}$. Possible infl points at $x = \underline{-100}$.

4.

point of dim returns at $x = \underline{22}$ rel max at $x = \underline{34.6}$.

other infl points at $x = \underline{0, 47.3}$ rel min at $x = \underline{90}$.

other possible infl points at $x = \underline{90}$ other critical points at $x = \underline{0}$.

5.

rel max at $x = \underline{0}$ | point of dim returns at $x = \underline{-1.43}$.

rel min at $x = \underline{4.29}$ | other infl points at $x = \underline{0}$.

other critical points at $x = \underline{\text{none}}$ | other possible infl points at $x = \underline{\text{none}}$

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6.

point of dim returns at $x =$ none . rel max at $x =$ 4 .
 other infl points at $x =$ 8 . rel min at $x =$ 20 .
 other possible infl points at $x =$ 20 . other critical points at $x =$ none .

7.

point of dim returns at $x =$ none . rel max at $x =$ 14.3 .
 other infl points at $x =$ 28.6 . rel min at $x =$ 20 .
 other possible infl points at $x =$ 20 . other critical points at $x =$ none .

8.

point of dim returns at $x =$ none . rel max at $x =$ 0 .
 other infl points at $x =$ 20 . rel min at $x =$ 8 .
 other possible infl points at $x =$ none . other critical points at $x =$ 20 .

9.

rel max at $x =$ 10 . point of dim returns at $x =$ 10 .
 rel min at $x =$ 9.52 . other infl points at $x =$ 8.33, 10.71 .
 other critical points at $x =$ 10 . other possible infl points at $x =$ 0 .

10.

rel max at $x =$ 90 point of dim returns: $x =$ 0, 47.3
 rel min at $x =$ 34.6 other infl points: $x =$ 22.0
 other crit points: $x =$ 0 other possible infl points: $x =$ 90

11.

rel max: $x =$ 2.67 point of dim returns: $x =$ none .
 rel min: $x =$ 0 other infl points: $x =$ 4 .
 other crit points: $x =$ 4 other possible infl points: $x =$ 0 .

12.

point of dim returns at $x =$ _____ . rel max at $x =$ _____ .
 other infl points at $x =$ _____ . rel min at $x =$ _____ .
 other possible infl points at $x =$ _____ . other critical points at $x =$ _____ .

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13.point of dim returns at $x = \underline{\hspace{2cm}}$. rel max at $x = \underline{\hspace{2cm}}$.other infl points at $x = \underline{\hspace{2cm}}$. rel min at $x = \underline{\hspace{2cm}}$.other possible infl points at $x = \underline{\hspace{2cm}}$. other critical points at $x = \underline{\hspace{2cm}}$.**14.**

$$\text{a. } H'(x) = 1.2x^2 - 30x + 221 \qquad H''(x) = 2.4x - 30 = 0$$

$$x = 30/2.4 = 12.5 \text{ yrs after 1980}$$

b.

$H(x) = +0.4x^3$ ----- looks like  so the inflection point is a point of least rapid increase rather than a point of diminishing returns

15.

$$\text{a. } P'(t) = 0.028t^3 + 1.5t^2 - 26t + 137 \qquad P''(t) = 0.084t^2 + 3t - 26 = 0$$

$$t = (-3 + \sqrt{(3^2 - 4 \cdot 0.084 \cdot -26)}) / (2 \cdot 0.084) = \underline{7.21079} - 42.925 \text{ degrees Celsius}$$

b.

$P(t) = +0.007t^4$ ---- looks like  so the point of diminishing

returns (most rapid rate of change, -28.0679) is left of the point of least rapid rate of change, 22.0679.

16.

$$f'(x) = 0.04x^3 + 400 = 0$$

$$x = (-400/0.04)^{1/3} \approx -21.54 \approx -22 \qquad \text{min}$$

17.

$$f'(x) = 0.03x^2 - 4 = 0$$

$$x = (4/0.03)^{1/2} \approx \pm 11.547 \approx \pm 12 \qquad -12 \text{ is max} \quad +12 \text{ is min}$$

18.

$$f'(x) = 200[x(0.7^x)\ln(0.7) + 1 \cdot (0.7^x)] = 0$$

$$x = -1/\ln(0.7) = 2.8037 \approx 3 \qquad \text{max}$$

19.

$$A' = 0.03t^2 - t + 10$$

$$A'' = 0.06t - 1 = 0$$

$$t = 1/0.06 = 16.667 \text{ years after 1970}$$