Absolute Extrema

5.5

Suppose f(x) is a continuous function whose domain is a closed interval [a, b]. Then we say f(x) attains an ***absolute maximum*** at x = c if, for all x in [a, b], f(x) $\leq$ f(c). Similarly, f has an ***absolute minimum*** at x = c if f(x) $\geq$ f(c) for all x in [a, b]. The function f(x) may attain the same maximum and minimum values elsewhere in the interval.

An important **theorem** tells us that must attain **any continuous function on a closed interval [a, b] must attain an absolute maximum (and an absolute minimum) somewhere in the interval**.

Another **theorem** tells us that **if a function f(x) is differentiable at all but finitely many points in [a, b] then the *absolute extrema* (maxima and minima) must be attained where f'(x) is zero or undefined.**

Thus, to find the maximum or minimum value of a reasonable function f(x)

1. We differentiate it.

2. We list the finitely many points where $f'(x)$ is undefined.

3. We solve the equation $f'(x) = 0$ to find the other critical points, typically just finitely many.

4. We compare the values of f(x) at these finitely many points.

Understand that, even though technology these days can compare many values of f(x) rapidly to approximate an extreme value, this brute force approach does not work with functions of more than one variable even as simple as volumes and areas.

In application, quantities such as the cost of a product or the market shared earned by a company are influenced by several variables and cannot even be in two or three dimensional graphs. One of the important about the methods of Calculus is that they do generalize to functions with several variables.

Also understand that students are usually given problems of analysis to practice, such as finding the zeros of a function or finding the maxima and minima. Jobs where one does this are sometimes described as “C student jobs” – somebody tells you what to do and you do it. These jobs do not earn a lot of money.

The employee who is worth more to an employer is one who can design processes so that the maxima and minima have certain characteristics. These people work with functions like $ax^3 + bx^2 + cx + d$, which a calculator cannot analyze with being given values for a, b and c.

These kinds of employees, who have done well in courses like managerial accounting instead of just analytical accounting, will be worth more to an employer and command a better salary.