

Chap 6 Sec 1 – 2

Calculate the following indefinite integrals

1. $\int x\sqrt{x+1}dx$

6. $\int 3x(x+5)^{17} dx$

2. $\int \frac{x}{\sqrt{x+1}} dx$

7. $\int \frac{7x}{\sqrt{x+5}} dx$

3. $\int \frac{x+2}{\sqrt{x-3}} dx$

8. $\int x\sqrt{2x+1}dx$

4. $\int \frac{2x-1}{\sqrt{x-3}} dx$

9. $\int 11x(2x-17)^5 dx$

5. $\int \frac{x}{\sqrt{2x-3}} dx$

10. $\int 17x(5x+11)^8 dx$

Answers:

1. $\int x\sqrt{x+1}dx = \int x(x+1)^{1/2} dx$

The composition suggests $u = x + 1$. Try it even it is hard to imagine how it can help. Then $x = u - 1$ and $du = dx$.

$$\begin{aligned} \int x(x+1)^{1/2} dx &= \int (u-1)u^{1/2} du = \int u^{3/2} - u^{1/2} du \\ &= \frac{u^{5/2}}{5/2} - \frac{u^{3/2}}{3/2} + c = \frac{(x+1)^{5/2}}{5/2} - \frac{(x+1)^{3/2}}{3/2} + c \end{aligned}$$

2. $\int \frac{x}{\sqrt{x+1}} dx : u = x + 1; du = dx; x = u - 1$

$$\int (u-1)u^{-1/2} du = \int u^{1/2} - u^{-1/2} du = \frac{(x+1)^{3/2}}{3/2} - \frac{(x+1)^{1/2}}{1/2} + c$$

3. $\int \frac{x+2}{\sqrt{x-3}} dx : u = x - 3; du = dx; x = u + 3$

$$\int \frac{x+2}{\sqrt{x-3}} dx = \int (u+5)u^{-1/2} du = \frac{(x-3)^{3/2}}{3/2} + 5 \frac{(x-3)^{1/2}}{1/2} + c$$

4. $\int \frac{2x-1}{\sqrt{x-3}} dx : u = x - 3; x = u + 3; du = dx$

$$\begin{aligned} \int \frac{2x-1}{\sqrt{x-3}} dx &= \int [2(u+3)-1]u^{-1/2} du = \int 2u^{1/2} + 5u^{-1/2} du \\ &= 2 \frac{(x-3)^{3/2}}{3/2} + 5 \frac{(x-3)^{1/2}}{1/2} + c \end{aligned}$$

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$$5. \int \frac{x}{\sqrt{2x-3}} dx : u = 2x - 3; du = 2dx; x = \frac{u+3}{2}$$

$$\int \frac{x}{\sqrt{2x-3}} dx = \int \frac{u+3}{2} u^{-1/2} \frac{du}{2} = \frac{1}{4} \int u^{1/2} + 3u^{-1/2} du$$

$$= \frac{1}{4} \left[\frac{(2x-3)^{3/2}}{3/2} + 3 \frac{(2x-3)^{1/2}}{1/2} \right] + C$$

$$6. \int 3x(x+5)^{17} dx : u = x + 5; x = u - 5; du = dx$$

$$\int 3x(x+5)^{17} dx = 3 \int (u-5)u^{17} du = 3 \int u^{18} - 5u^{17} du = 3 \left[\frac{(x+5)^{19}}{19} + \frac{5(x+5)^{18}}{18} \right] + C$$

$$7. \int \frac{7x}{\sqrt{x+5}} dx : u = x + 5; x = u - 5; du = dx$$

$$\int \frac{7x}{\sqrt{x+5}} dx = 7 \int (u-5)u^{-1/2} du = 7 \int u^{1/2} - 5u^{-1/2} du = 7 \left[\frac{(x+5)^{3/2}}{3/2} - 5 \frac{(x+5)^{1/2}}{1/2} \right] + C$$

$$8. \int x\sqrt{2x+1} dx : u = 2x + 1; x = \frac{u-1}{2}; du = 2dx; dx = \frac{du}{2}$$

$$\int x\sqrt{2x+1} dx = \int \frac{u-1}{2} u^{1/2} \frac{du}{2} = \frac{1}{2^2} \int u^{3/2} - u^{1/2} du$$

$$= \frac{1}{2^2} \left[\frac{(2x+1)^{5/2}}{5/2} - \frac{(2x+1)^{3/2}}{3/2} \right] + C$$

$$9. \int 11x(2x-17)^5 dx : u = 2x - 17; x = \frac{u+17}{2}; du = 2dx; dx = \frac{du}{2}$$

$$\int 11x(2x-17)^5 dx = 11 \int \frac{u+17}{2} \cdot u^5 \frac{du}{2} = \frac{11}{2^2} \int u^6 + 17u^5 du$$

$$= \frac{11}{2^2} \left[\frac{(2x-17)^7}{7} + \frac{17(2x-17)^6}{6} \right] + C$$

$$10. \int 17x(5x+11)^8 dx : u = 5x + 11; x = \frac{u-11}{5}; du = 5dx; dx = \frac{du}{5}$$

$$\int 17x(5x+11)^8 dx = 17 \int \frac{u-11}{5} \cdot u^8 \frac{du}{5} = \frac{17}{5^2} \int u^9 - 11u^8 du$$

$$= \frac{17}{5^2} \left[\frac{(5x+11)^{10}}{10} - \frac{11(5x+11)^9}{9} \right] + C$$