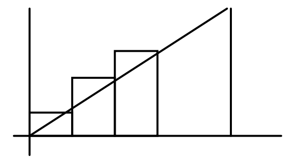
Supplementary Problems

Instr Jamieson

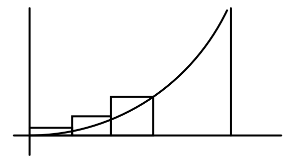
Ch. 6 Sec 5

$$\sum_{i=1}^{n} i = \frac{n(n+1)}{2} \qquad \sum_{i=1}^{n} i^2 = \frac{n(n+1)(2n+1)}{6} \qquad \sum_{i=1}^{n} i^3 = \frac{n^2(n+1)^2}{4}$$

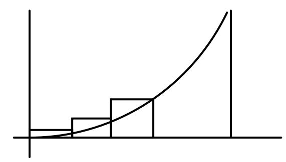
1. Beginning with a sum of areas of n approximating rectangles, find the limit of the Riemann sum for the area under f(x) = 7x from x = 0 to x= 13.



2. Beginning with a sum of areas of n approximating rectangles, find the limit of the Riemann sum for the area under f(x) = $7x^2$ from x = 0 to x= 13.



3. Beginning with a sum of areas of n approximating rectangles, find the limit of the Riemann sum for the area under f(x) = $7x^3$ from x = 0 to x= 13.



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Answers:

1. $\frac{13-0}{n}\cdot\left(7\cdot 1\frac{13}{n}\right)+\frac{13-0}{n}\cdot\left(7\cdot 2\frac{13}{n}\right)+\frac{13-0}{n}\cdot\left(7\cdot 3\frac{13}{n}\right)+\cdots\frac{13-0}{n}\cdot\left(7\cdot n\frac{13}{n}\right)$

$=\frac{13}{n}\cdot\frac{13}{n}\cdot 7\cdot[1+2+3+\cdots+n]$

$=\frac{13}{n}\cdot\frac{13}{n}\cdot 7\cdot\frac{n(n+1)}{2}$

$=\frac{7\cdot 13^2}{2}\frac{n+1}{n}\to\frac{7\cdot 13^2}{2}$ as $n\to\infty$

2. $\frac{13-0}{n}\cdot\left(7\cdot 1\frac{13}{n}\right)^2+\frac{13-0}{n}\cdot\left(7\cdot 2\frac{13}{n}\right)^2+\frac{13-0}{n}\cdot\left(7\cdot 3\frac{13}{n}\right)^2+\cdots\frac{13-0}{n}\cdot\left(7\cdot n\frac{13}{n}\right)^2$

$=7\cdot\frac{13}{n}\left(\frac{13}{n}\right)^2\left[1^2+2^2+3^2+\cdots+n^2\right]$

$=7\cdot\left(\frac{13}{n}\right)^3\frac{n(n+1)(2n+1)}{6}$

$=\frac{7\cdot 13^3}{6}\frac{(n+1)(2n+1)}{n^2}=\frac{7\cdot 13^3}{6}\left(\frac{2n^2+3n+1}{n^2}\right)\to\frac{7\cdot 13^3}{3}$ as $n\to\infty$

3. $\frac{13-0}{n}\cdot\left(7\cdot 1\frac{13}{n}\right)^3+\frac{13-0}{n}\cdot\left(7\cdot 2\frac{13}{n}\right)^3+\frac{13-0}{n}\cdot\left(7\cdot 3\frac{13}{n}\right)^3+\cdots\frac{13-0}{n}\cdot\left(7\cdot n\frac{13}{n}\right)^3$

$=7\cdot\frac{13}{n}\left(\frac{13}{n}\right)^3\left[1^3+2^3+3^3+\cdots+n^3\right]$

$=7\cdot\left(\frac{13}{n}\right)^4\frac{n^2(n+1)^2}{4}$

$=\frac{7\cdot 13^4}{4}\frac{(n+1)^2}{n^2}=\frac{7\cdot 13^4}{4}\left(\frac{n^2+2n+1}{n^2}\right)\to\frac{7\cdot 13^4}{4}$ as $n\to\infty$