

Test Solutions

Ch 4.3 – 4, 3.6 – 7, 5.1 – 3

Name: _____

There are 27 points on this test.

$f(x)$	$f'(x)$
u^n	$nu^{n-1}u'$
au	au'
a	0
e^u	$e^u u'$
a^u	$\ln(a) \cdot a^u u'$
$\ln(u)$	$\frac{1}{u} \cdot u'$
$\log_a(u)$	$\frac{1}{\ln(a)u} \cdot u'$
$a \cdot u(x) \pm b \cdot v(x)$	$a \cdot u' \pm b \cdot v'$
$a \cdot u(x)$	$a \cdot u'$
$u(x) \cdot v(x)$	$u \cdot v' + v \cdot u'$
$\frac{u}{v}$	$\frac{v \cdot u' - u \cdot v'}{v^2}$

$f(x)$	$F(x) = \int f(x)dx$
$u^n \cdot u'$	$\frac{u^{n+1}}{n+1} + C, n \neq -1$
$\frac{1}{u} \cdot u'$	$\ln(u) + C$
a	$ax + C$
$e^u \cdot u'$	$e^u + C$
$a^u \cdot u'$	$\frac{a^u}{\ln(a)} + C$

$$A_0 e^{rt}$$

$$A_0 \left(1 + \frac{r}{n}\right)^{nt}$$

$$\frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

In #1 – 5 differentiate the given function

1. (2 pt) $f(x) = e^{7x} \ln(x-5)$

$$f'(x) = e^{7x} \cdot 7 \cdot \ln(x-5) + e^{7x} \frac{1}{x-5} \quad (-1 \text{ per detail; } -2 \text{ if it does not have structure of the product rule})$$

2. (3 pt) $f(x) = e^{9x} \ln((x+1)^4)$ (-1 per detail; -2 if it does not have correct order of operations)

$$f'(x) = e^{9x} \cdot 9 \cdot \ln((x+1)^4) + e^{9x} \frac{1}{(x+1)^4} \cdot 4(x+1)^3$$

3. (3 pt) $f(x) = \ln((x+1)^3(x+2)^5)$ (-1 per detail; -2 if it does not have correct order of operations)

$$f'(x) = \frac{1}{(x+1)^3(x+2)^5} \left[3(x+1)^2(x+2)^5 + (x+1)^3 \cdot 5(x+2)^4 \right]$$

4. (2 pt) $f(x) = \frac{280}{1+5e^{-0.03x}}$ [Simplify your answer.]

(-1 per detail, including simplification; -2 if it does not have structure of the quotient rule)

$$f'(x) = \frac{(1+5e^{-0.03x}) \cdot 0 - 280 \cdot 5e^{-0.03x} \cdot (-0.03)}{(1+5e^{-0.03x})^2}$$

$$= \frac{-280 \cdot 5e^{-0.03x} \cdot (-0.03)}{(1+5e^{-0.03x})^2}$$

5. (3 pt) $f(x) = e^{3x} \cdot 2^x \cdot 1.03^x$ (-1 per detail; -2 if it does not have correct order of operations)

$$f'(x) = e^{3x} \cdot 3 \cdot 2^x \cdot 1.03^x + e^{3x} \cdot 2^x \ln(2) \cdot 1.03^x + e^{3x} \cdot 2^x \cdot 1.03^x \ln(1.03)$$

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6. (2 pt) Our company's profit is $P(x) = -0.3x^3 - 30x^2 + 100x + 1000$ dollars when we produce x pounds per month of our commodity. We are currently producing 30 pounds per month. Use marginal analysis to estimate the additional profit earned if we increase production by 0.2 pounds per month.

Answer: -502

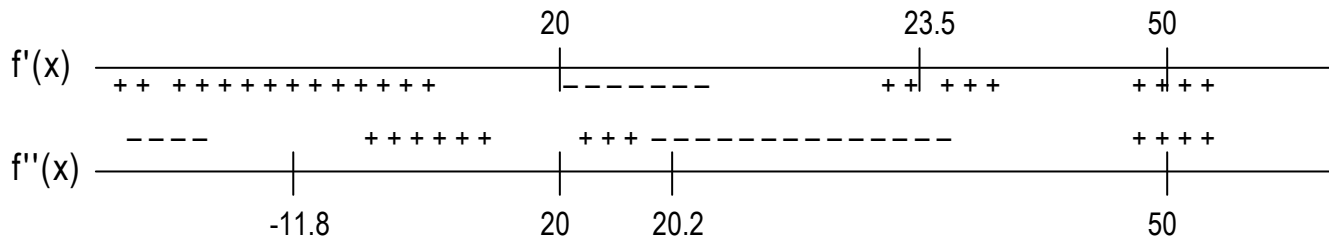
$$P'(x) = -0.9x^2 - 60x + 100$$

$$P(30) = -2510 \quad (1 \text{ pt})$$

$$P(30) \cdot 0.2 = -502 \quad (1 \text{ pt})$$

7. (4 pt) For $f(x) = 100(x - 20)^{2/5}(x - 50)^3$

$$f'(x) = 340(x - 20)^{-3/5}(x - 50)^2[x - 23.5] \text{ and } f''(x) = 816(x - 20)^{-8/5}(x - 50)(x + 11.8)(x - 20.2)$$



The critical points and the possible inflection points are those shown with the appropriate derivative. Classify them.
 (1 + 1 left column) (1 + 1 right column)

PDR at $x =$ 20.2 Min at $x =$ 23.5

Other inflection points at $x =$ -11.8, 50 Max at $x =$ 20

Other possible inflection points at $x =$ 20 Other critical points at $x =$ 50

8. (3 pt) A company's profit $P(x) = (-500,000 + 7000x)e^{-0.03x}$ dollars is determined by its sales x . Find the sales level that maximizes profit. Round to the nearest unit.

$$x = 104.76$$

$$P'(x) = 7000e^{-0.03x} + (-500,000 + 7000x)e^{-0.03x}(-0.03) = 0 \quad (1 \text{ pt})$$

$$7000 + (-500,000 + 7000x)(-0.03) = 0 \quad (1 \text{ pt})$$

$$x = (-7000/-0.03 + 500000)/7000 = 104.76 \quad (1 \text{ pt})$$

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9. (3 pt) Find and classify the inflection points of $f(x) = -0.1x^4 + x^3 + 0.2x^2 + 3x + 5$. Justify the classification. Use three significant digits precision. (1 pt for classification)

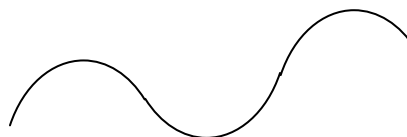
$$f'(x) = -0.4x^3 + 3x^2 + 0.4x + 3$$

PDR at $x = 5.0658$

$$f''(x) = -1.2x^2 + 6x + 0.4 = 0 \quad (1 \text{ pt})$$

Other inflection point at $x = -0.0658$ Other possible inflection point at $x =$

$$x = (-6 \pm \sqrt{6^2 - 4 \cdot (-1.2) \cdot 0.4}) / (-2.4) = -0.0658, 5.0658 \quad (1 \text{ pt})$$



10. (4 pt) Find and classify the critical points of $f(x) = 0.2x^4 + 4x^3 - 0.8x^2 + 5$. Justify the classification. Use three significant digits precision. (1 pt for classification)

$$f'(x) = 0.8x^3 + 12x^2 - 1.6x = 0$$

Max at $x = 0$

$$x(0.8x^2 + 12x - 1.6) = 0 \quad (1 \text{ pt})$$

Min at $x = 0.132, -15.132$ Other critical points at $x =$

(1 pt)

$$x = (-12 \pm \sqrt{12^2 - 4 \cdot 0.8 \cdot (-1.6)}) / 1.6 = 0.132, -15.132$$

or

$$x = 0$$

(1 pt for remembering 0)

