MAC 2233.30 — 2011 Summer B

Test: Ch. 3.1 – 3 & Ch. 4.2 & 4 Solutions

Instr: Jamieson

Name: ______________________________

There are 31 points on this test.

For any functions u(x) and v(x) and any constants a, b and n the following table applies.

$f(x)$	$f'(x)$
u^n	$nu^{n-1}\cdot u'$
$a\cdot u$	$a\cdot u'$
a	0
e^u	$e^u\cdot u'$
a^u	$\ln(a)\cdot a^u\cdot u'$
$\ln(u)$	$\frac{1}{u}\cdot u'$
$\log_a(u)$	$\frac{1}{\ln(a)u}\cdot u'$
$a\cdot u(x)\pm b\cdot v(x)$	$a\cdot u'\pm b\cdot v'$
$a\cdot u(x)$	$a\cdot u'$
$u(x)\cdot v(x)$	$u\cdot v'+v\cdot u'$
$\frac{u}{v}$	$\frac{v\cdot u'-u\cdot v'}{v^2}$

$f(x)$	$\mathbf{F(x)=\int f(x)dx}$
$u^n\cdot u'$	$\frac{u^{n+1}}{n+1}+C, n\neq -1$
$\frac{1}{u}\cdot u'$	$\ln(\lvert u\rvert)+C$
a	$ax+C$
$e^u\cdot u'$	e^u+C
$a^u\cdot u'$	$\frac{a^u}{\ln(a)}+C$

$$A_0e^{rt}$$

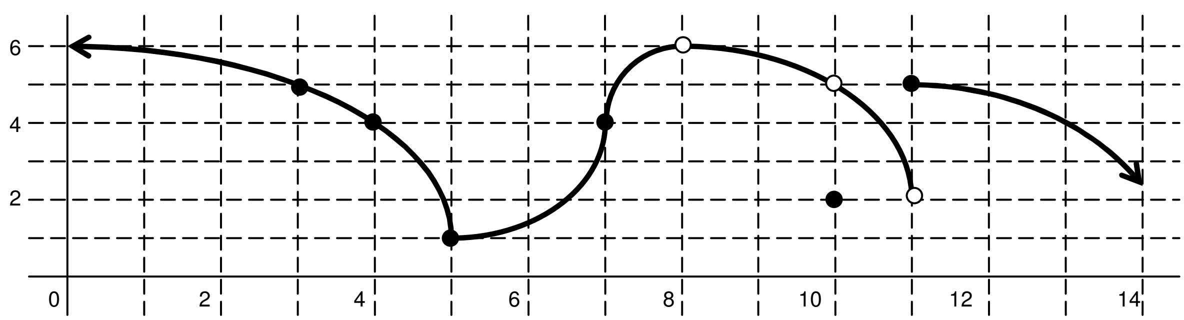
$$A_0\left(1+\frac{r}{n}\right)^{nt}$$

$$\frac{-b\pm\sqrt{b^2-4ac}}{2a}$$

1. All answers in this problem are integers. The function shown is

(1 pt) discontinuous at x = 8, 10, 11

(1 pt) non-differentiable at x = 8, 10, 11, 5, 7



(1 pt) $\lim_{x\to 8} f(x) =$ 6 $\lim_{x\to 11^-} f(x) =$ 2 $\lim_{x\to 11} f(x) =$ undefd

$f(10) =$ 2 $f(11) =$ 5

Test: Ch. 3.1 – 3 & Ch. 4.2 & 4 Solutions

2. For $f(x)=\begin{cases} x^2+6 & ,\text{if } x<2 \\ 10 & ,\text{if } x=2 \\ 14-2x & ,\text{if } x>2 \end{cases}$

(1 pt) $\lim_{x\to 2^-} f(x) =$ 10 and $\lim_{x\to 2} f(x) =$ 10

(1 pt) Is f(x) continuous at x = 2? Circle YES or NO.

3. (2 pt) Suppose f(x) and g(x) are continuous and increasing functions for all x with integer limits at x = 7. Give these limits.

x	**f(x)**	**g(x)**
6.8	4.765	15.2
6.9	5.027	15.7
6.99	5.971	15.89
6.999	5.996	15.98
7.001	6.016	16.06
7.01	6.027	16.17
7.1	6.169	16.89
7.1	7.002	17.62

$\lim_{x\to 7} f(x) =$ 6

$\lim_{x\to 7} [f(x)\cdot g(x)] =$ 6·16 = 96 (–1 for each mistake)

$\lim_{x\to 7} [3\cdot f(x)] =$ 3·6 = 18

$\lim_{x\to 7} \sqrt{g(x)} =$ √(16) = 4

In #4 – 5 find and simplify $f''(x)$ for the given function. Circle your answer.

4. (2 pt) $f(x)=\dfrac{(x+3)(x-2)}{x}+\sqrt{7}\cdot e^x-\dfrac{e^2+1}{\pi}+\ln\left(e^2+3\right)$

$$f(x)=\frac{x^2+x-6}{x}+\sqrt{7}\cdot e^x-\frac{e^2+1}{\pi}+\ln\left(e^2+3\right)$$

$$f(x)=x+1-6x^{-1}+\sqrt{7}\cdot e^x-\frac{e^2+1}{\pi}+\ln\left(e^2+3\right)$$ (1 pt)

$$f'(x)=1+6x^{-2}+\sqrt{7}\cdot e^x$$

$$f''(x)=-12x^{-3}+\sqrt{7}\cdot e^x$$ (1 pt)

5. (2 pt) $f(x)=\dfrac{x^2+7x^4-16\sqrt{x}}{x^2}$

$$f(x)=\frac{x^2+7x^4-16x^{1/2}}{x^2}$$

$$f(x)=1+7x^2-16x^{-3/2}$$ (1 pt)

$$f'(x)=14x+16\cdot\frac{3}{2}\cdot x^{-5/2}$$

$$f''(x)=14-16\cdot\frac{3}{2}\cdot\frac{5}{2}\cdot x^{-5/2}=14-60x^{-5/2}$$ (1 pt)

In #6 – 10 differentiate each function.

6. (2 pt) $f(x)=\left(7x^3-5\right)^{13}$

$$\boxed{13\left(7x^3-5\right)^{12}}\cdot\boxed{21x^2}$$

(1 pt) (1 pt)

7. (3 pt) $f(x)=\left(7(2x+1)^3-5\right)^{13}$

$$\boxed{13\left(7(2x+1)^3-5\right)^{12}}\cdot\boxed{21(2x+1)^2}\cdot\boxed{2}$$

(1 pt) (1 pt) (1 pt)

Test: Ch. 3.1 – 3 & Ch. 4.2 & 4
Solutions

8. (3 pt) $f(x) = \ln\left(\left(x^7 + 2\right)^5 + 9\right)$

$$\boxed{\frac{1}{\left(x^7+2\right)^5+9}} \cdot \boxed{5\left(x^7+2\right)^4} \cdot \boxed{7x^6}$$

(1 pt) (1 pt) (1 pt)

9. (3 pt) $f(x) = 2\left(3x + 5e^x\right)^7$

$$\boxed{14\left(3x+5e^x\right)^6} \cdot \boxed{\left(3+5e^x\right)}$$

(1 pt) (1 pt for “3 + 5x” and 1 pt for <u>parentheses</u>)

MAC 2233.30 — *Test: Ch. 3.1 – 3 & Ch. 4.2 & 4* — Instr: Jamieson

2011 Summer B — *Solutions*

10. (4 pt) Using the definition, $\frac{df}{dx} = \lim_{h\to 0}\left(\frac{f(x+h)-f(x)}{h}\right)$ find $f'(x)$ for $f(x) = \frac{8}{7x-6}$ if $x \neq 6/7$.

$$\frac{\frac{8}{7(x+h)-6}-\frac{8}{7x-6}}{h}$$

$$\frac{8(7x-6)-8(7x+7h-6)}{h(7x+7h-6)(7x-6)}$$ (1 pt)

$$\frac{-8\cdot 7h}{h(7x+7h-6)(7x-6)}$$ (1 pt)

$$\frac{-8\cdot 7}{(7x+7h-6)(7x-6)}$$ (1 pt)

$$\lim_{h\to 0}\left[\frac{-8\cdot 7}{(7x+7h-6)(7x-6)}\right] = \frac{-8\cdot 7}{(7x-6)(7x-6)} = \frac{-8\cdot 7}{(7x-6)^2}$$ (1 pt)

Test: Ch. 3.1 – 3 & Ch. 4.2 & 4 Solutions

11. (5 pt) Using the definition, $\frac{df}{dx} = \lim_{h \to 0}\left(\frac{f(x+h)-f(x)}{h}\right)$ find $f'(x)$ for $f(x) = 6\sqrt{3x+7}$ if x ≠ -7/3.

$$\frac{6\sqrt{3(x+h)+7} - 6\sqrt{3x+7}}{h} \cdot \frac{6\sqrt{3(x+h)+7} + 6\sqrt{3x+7}}{6\sqrt{3(x+h)+7} + 6\sqrt{3x+7}}$$ (1 pt)

$$\frac{6(3x+3h+7) - 6(3x+7)}{h(6\sqrt{3(x+h)+7} + 6\sqrt{3x+7})}$$ (1 pt)

$$\frac{6 \cdot 3h}{h(6\sqrt{3(x+h)+7} + 6\sqrt{3x+7})}$$ (1 pt)

$$\frac{6 \cdot 3}{6\sqrt{3(x+h)+7} + 6\sqrt{3x+7}}$$ (1 pt)

$$\lim_{h \to 0}\left[\frac{6 \cdot 3}{6\sqrt{3(x+h)+7} + 6\sqrt{3x+7}}\right] = \frac{6 \cdot 3}{6\sqrt{3x+7} + 6\sqrt{3x+7}} = \frac{6 \cdot 3}{12\sqrt{3x+7}},$$ if x ≠ -7/3. (1 pt)