MAC 2233

Chap 4.5 – 6 and Chap 6.1 – 5 Test Soln

Instr: Jamieson

Name: ______________________________

There are 27 points on this test. Show work where indicated.

f(x)	**f'(x)**
u^n	$nu^{n-1}u'$
au	au'
a	0
e^u	$e^u u'$
a^u	$\ln(a)\cdot a^u u'$
$\ln(u)$	$\frac{1}{u}\cdot u'$
$\log_a(u)$	$\frac{1}{\ln(a)u}\cdot u'$
$a\cdot u(x)\pm b\cdot v(x)$	$a\cdot u'\pm b\cdot v'$
$a\cdot u(x)$	$a\cdot u'$
$u\cdot v$	$u\cdot v'+v\cdot u'$
$\frac{u}{v}$	$\frac{v\cdot u'-u\cdot v'}{v^2}$

f(x)	$\mathbf{F(x)=\int f(x)dx}$
$u^n\cdot u'$	$\frac{u^{n+1}}{n+1}+C, n\neq -1$
$\frac{1}{u}\cdot u'$	$\ln(\lvert u\rvert)+C$
a	$ax+C$
$e^u\cdot u'$	e^u+C
$a^u\cdot u'$	$\frac{a^u}{\ln(a)}+C$

$$A_0e^{rt}$$

$$A_0\left(1+\frac{r}{n}\right)^{nt}$$

$$\frac{-b\pm\sqrt{b^2-4ac}}{2a}$$

1. (2 points) Calculate $\int \frac{x^2-3x+7\sqrt{x}}{x^2}dx$

$$\int 1-\frac{3}{x}+7x^{-1.5}dx = x-3\ln(\lvert x\rvert)+7\frac{x^{-0.5}}{-0.5}+C$$

(1 pt) (1 pt)

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In #2 – 6 circle one of the integrals that is beyond the scope of this course (1 pt) and evaluate the other(s).

2. (3 points)

a. $\int 7\left(1.03^x\right)dx$

$$\frac{7\left(1.03^x\right)}{\ln(1.03)}+C$$

(1 pt)

[✓ **b.**] $\int \ln(x)dx$

(1 pt)

c. $\int 10x^4(2x^5-9)^3dx$

$$\frac{(2x^5-9)^4}{4}+C$$

(1 pt)

3. (5 points)

[✓ **a.**] $\int 5x^4\left(x^3+1\right)^5dx$

(1 pt)

b. $\int 7x^2(2x^3-9)^7dx$

$$\frac{7}{6}\int 6x^2(2x^3-9)^7dx$$

$$\frac{7}{6}\frac{(2x^3-9)^8}{8}+C$$

(1 + 1)

c. $\int 11(3x-7)^9dx$

$$\frac{11}{3}\int 3(3x-7)^9dx$$

$$\frac{11}{3}\frac{(3x-7)^{10}}{10}+C$$

(1 + 1)

4. (5 points)

a. $\int \frac{e^x}{1+e^x}dx$

$$\ln\left(1+e^x\right)+C$$

(1 +1)

b. $\int \frac{2x}{\left(x^2+1\right)^5}dx$

$$\int 2x\left(x^2+1\right)^{-5}dx$$

$$\frac{\left(x^2+1\right)^{-4}}{-4}+C$$ (1 +1)

[✓ **c.**] $\int xe^xdx$

(1 pt)

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5. (4 points) Calculate $\int \frac{x}{\sqrt{2x-3}}dx$

$$u = 2x - 3 \qquad x = \frac{u+3}{2}$$

$$du = 2dx \qquad dx = \frac{du}{2} \qquad \text{(1 pt)}$$

$$\int \frac{u+3}{2} \cdot u^{-1/2} \frac{du}{2} \qquad \text{(1 pt)}$$

$$\frac{1}{2^2}\int u^{1/2} + 3u^{-1/2} du$$

$$\frac{1}{2^2}\left[\frac{u^{3/2}}{3/2} + 3\frac{u^{1/2}}{1/2}\right] + C \qquad \text{(1 pt)}$$

$$\frac{1}{2^2}\left[\frac{(2x-3)^{3/2}}{3/2} + 3\frac{(2x-3)^{1/2}}{1/2}\right] + C \qquad \text{(1 pt)}$$

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6.

a. (2 points) Solve $\frac{dy}{dt} - \frac{6t^5}{5+6y} = 0$ implicitly as R(y,t) = 0.

$$\frac{dy}{dt} = \frac{6t^5}{5+6y}$$

$(5+6y)dy = 6t^5 dt$ (1 pt)

$5y + 3y^2 = t^6 + C$ (1 pt)

b. (1 point bonus) Give an explicit solution y(t) =

$$3y^2 + 5y - \left(t^6 + C\right) = 0$$

$y = \frac{-5 \pm \sqrt{5^2 + 4 \cdot 3 \cdot \left(t^6 + C\right)}}{2 \cdot 3}$ (1 pt)

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7. (2 points) Find $\frac{dx}{dt}$ exactly when x = 3, y = 2 and $\frac{dy}{dt} = -4$ if x(t) and y(t) are differentiable functions which satisfy $3x + 2y + y^3 = 21$

$\frac{dx}{dt} = \frac{56}{3}$ (1 pt)

$3x' + 2y' + 3y^2 y' = 0$ (1 pt)

$3x' + 2(-4) + 3(2)^2(-4) = 0$

3x' = 56

8. (3 points) Find $\frac{dy}{dt}$ exactly when t = 2 and y = 1, if y(t) is a differentiable function which satisfies

$3e^{2t} + 2t^2 y + y^3 = 3e^2 + 9$

$\frac{dy}{dt} = \frac{-6e^4 - 8}{11}$ (1 pt)

$3e^{2t}(2) + 2\left[t^2 y' + 2ty\right] + 3y^2 y' = 0$ (2 pt)

$3e^{2(2)}(2) + 2\left[2^2 y' + 2ty\right] + 3(1)^2 y' = 0$

$11y' = -6e^4 - 8$